Analysis of Spin Financial Market by GARCH Model

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Introduction

Stylized properties of asset returns

- Absence of autocorrelations
- Slow decay of autocorrelation in absolute returns
- Fat-tailed (heavy tail) distributions
- Volatility clustering
- Leverage effect
- Volume/volatility correlation
- …..

Volatility models

- ARCH
- GARCH
- EGARCH
- QGARCH
- GJR-GARCH
- SV model
- …

These models are used for volatility forecasts
FIG. 3. (a) Semilog plot of the autocorrelation function for the S&P 500 returns $G_{\Delta t}(t)$ sampled at a $\Delta t = 1$ min time scale,

$$C_{\Delta t}(\tau) = \frac{\langle G_{\Delta t}(t)G_{\Delta t}(t+\tau) \rangle - \langle G_{\Delta t}(t) \rangle^2}{\langle (G_{\Delta t}(t))^2 \rangle - \langle G_{\Delta t}(t) \rangle^2}.$$ 

Gopikrishnan et. al(1999)
Fat-tailed return distribution

Gopikrishnan et al., cond-mat/9905305
Volatility clustering and price return

Gopikrishnan et al., cond-mat/9905305
Spin financial models

Argents are assigned to spins

A dynamics is introduced and spins interact each other

Various models have been proposed

- Cont and Bouchaud (2000)
- Stauffer and Penna (1998)
- Iori (1999)
- Sznajd-Weron and Weron (2002)
- Sanchez (2002)
- Bornholdt (2001)
- Takaishi (2005)
- etc

These models appear to show some of the stylized facts

- Volatility clustering
- Fat-tailed distribution
- etc

Volatility property?
We study volatility property by GARCH model
Spin model

**Two states spin model**


Agents live at sites on an n-dimensional lattice.
(In this study we use a 2-dimensional lattice.)

Each site has a spin.

\[ S_i \text{ takes } +1 \text{ or } -1 \]

- **Buy**
- **Sell**

We may assign +1 state to “Buy order” and -1 state to “Sell order”
$$M(t) = \frac{1}{n} \sum_{j} S_j(t)$$

Difference between buy and sell orders

$$h_i(t) = \sum_{j=1}^{n} J_{ij} S_j(t) - \alpha S_i(t) |M(t)|$$

Local interaction

Global interaction

$$\alpha > 0$$

Spins are updated by the following probability

$$S_i(t + 1) = +1 \quad p = 1/(1 + \exp(-2\beta h_i(t)))$$

$$S_i(t + 1) = -1 \quad 1 - p$$

Local interaction: Majority effect

Global interaction: Minority effect
Three states model (Potts-like model)

$$S_i$$ takes +1, -1 or 0

- Buy
- Sell
- Inactive

We may assign +1 state to "Buy order" and -1 state to "Sell order"
0 state to "Inactive"

$$\gamma K$$ K: the number of inactive states
Simulation results (two state model)

\[ M(t) = \frac{1}{n} \sum_{j} S_j(t) \]

\[ L=100 \quad \beta=2 \quad \alpha=20 \]
\[ r(t) = \frac{[M(t) - M(t-1)]}{2} \]

Return time series

Volatility clustering
Return distributions

$L=100 \quad \beta=2$

\(\alpha=05\)
\(\alpha=10\)
\(\alpha=15\)
\(\alpha=20\)
\(\alpha=30\)
Cumulative return distributions

$L=100 \quad \beta=2$

$r^{-2.2}$

- $\alpha=20$
- $\alpha=10$
- $\alpha=0.5$
GARCH model


time series $r_t$

$$r_t = \sigma_t \varepsilon_t,$$

$$\varepsilon_t \sim N(0, 1)$$

Gaussian noise with mean 0 and variance 1

$\sigma_t$: volatility

Volatility process

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2$$

$\alpha, \beta, \gamma$ and $\omega$ are model parameters we have to determine

We use the Bayesian inference by the Markov Chain Monte Carlo method to determine the model parameters
$L=100 \times 100, \quad \beta=4.0, \quad \alpha=10, \quad \gamma=1$

Determine GARCH parameters from these data.
Volatility time series from GARCH model

<table>
<thead>
<tr>
<th>α</th>
<th>β</th>
<th>ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.050</td>
<td>0.949</td>
<td>2.69x10^{-9}</td>
</tr>
</tbody>
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\[ \alpha + \beta \approx 0.999 \]

This is also observed in real markets.
Comparison with empirical results


Empirical volatility from three measures

\[ \alpha + \beta \approx 0.984 \]
Conclusion

- Spin-based financial models can reproduce some stylized facts of financial markets.
- To study the volatility property we have applied the GARCH model for the data generated by the spin model.
- The time series data from the spin model shows the volatility clustering.
- We performed the Bayesian inference of the GARCH model for the data from the spin model and found that the GARCH parameters obtained are similar to those from the empirical data. Thus the volatility of the spin model has the similar persistency for the volatility clustering.
References

Realized volatility in Spin model

$L = 125 \times 125$, $\beta = 2.0$, $\alpha = 20$

We define $t = 1$ corresponds to one spin update.

$T = 125 \times 125 = 15625$
Realized volatility $dt=1$
Return distribution

\( r_t \)

Kurtosis: 43.2
Std. dev.: 0.00059

Sampling frequency \( dt=1 \)

\( \frac{r_t}{\sigma_t} \)

Kurtosis: 2.92
Std. dev.: 0.996
Variance of Standardized returns

- Variance of Standardized returns
  - dt
  - Variance
  - dt

Graph showing the variance of standardized returns over time.
Kurtosis of standardized returns

\[ A_0 \left(1 - \frac{2}{(125*125)/dt + 2}\right) \]
Spin model

\[ \left| \frac{r_t}{\sigma_t} \right| \]