PRINCIPAL COMPONENT ANALYSIS AND SYSTEMIC RISK IN THE JAPANESE STOCK MARKET

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Introduction

- We have experienced the Financial Crisis of 2007-2009.
- In financial crises the risk quickly propagates across the whole financial system. The risk associated with the whole financial system is called the systemic risk. When the systemic risk is high, the financial system is unstable.
- How can we detect such risk in the financial system?
- In financial crisis stocks collectively move and they are correlated each other. When many stocks are correlated (interconnected) the risk may easily spread over the system. So the correlations among stocks are important to see the risk. The correlations can be measured by the cross correlation matrix.
- In this study we calculate the cross correlations among stocks and use the principal component analysis to estimate systemic risk or system fragility in the Japanese stock market.
When many stocks are interconnected, the system is unstable.

When the interconnectivity is large, risk can easily spread over the system.

The interconnectivity is important to see if the system is unstable.

Information on the interconnectivity is given by the cross correlation matrix.

We calculate the cross correlation matrix and use the principal component analysis to extract a risk measure from the cross correlation matrix.
Principal component analysis

- Consider $N$ stocks, each consists of $T$ returns.

$$r_i(t) = \ln p_i(t) - \ln p_i(t-1) \quad i:1,\ldots,N \quad t:1,\ldots,T \quad p_i(t):\text{price}$$

- Construct cross correlation matrix

$$C_{ij} = \frac{E((r_i(t) - \bar{r}_i)(r_j(t) - \bar{r}_j))}{\sigma_i \sigma_j} \quad C_{ij} = \begin{pmatrix} 1 & C_{12} & \ldots & C_{1N} \\ C_{21} & 1 & \vdots \\ \vdots & \ddots & 1 \\ C_{N1} & \ldots & 1 \end{pmatrix} \quad C_{ij} = C_{ji}$$

$$\bar{r}_i = E(r_i(t))$$
Principal component analysis

- Eigenvalues of the cross correlation matrix

\[ \lambda_i : \lambda_1 > \lambda_2 > \cdots > \lambda_N \]

3D scatter plot of stocks A, B and C

The first vector is the one with the greatest variance in this direction.

The variance in this direction is given by the eigenvalue.

\[ \sigma_1^2 = \lambda_1 \]
Principal component analysis

- Total system variance
  \[ \sigma_s^2 = \sum_{j=1}^{N} \lambda_j = \text{Trace}[C] = N \]

- Define cumulative risk fraction (CRF) as a risk indicator
  \[ \text{CRF} (k) = \frac{\sum_{j=1}^{k} \lambda_j}{\sum_{j=1}^{N} \lambda_j} = \frac{\sum_{j=1}^{k} \lambda_j}{\text{Trace}[C]} = \frac{\sum_{j=1}^{k} \lambda_j}{N} \]

The CRF is the system variance with the first k principal components to the total variance.

If the system variance with the first few principal components is large, the financial system risk is expected to be high.

We try to see the system fragility by measuring the CRF.
DATA

• 366 stocks on the Tokyo Stock Exchange chosen from the companies listed on Topix 500
• From 01/05/1998 to 12/30/2013
• 3932 trading days
Stylized properties of asset returns

- Absence of autocorrelations
- Slow decay of autocorrelation in absolute returns
- Fat-tailed (heavy tail) distributions
- Volatility clustering
- Leverage effect
- Volume/volatility correlation
- .....
FIG. 3. (a) Semilog plot of the autocorrelation function for the S&P 500 returns $G_{\Delta t}(t)$ sampled at a $\Delta t = 1$ min time scale,

$$C_{\Delta t}(\tau) \equiv \frac{\langle G_{\Delta t}(t) G_{\Delta t}(t+\tau) \rangle - \langle G_{\Delta t}(t) \rangle^2}{\langle (G_{\Delta t}(t))^2 \rangle - \langle G_{\Delta t}(t) \rangle^2}.$$ 

(b) Absolute value of price returns

Gopikrishnan et. al (1999)
FIG. 4. (a) Loglog plot of the cumulative distribution of the normalized 1 min returns for the S&P 500 index.
High-Frequency Returns in 2001

10 NYSE stocks

Student-t distribution fits well to probability densities

Tsallis and Anteneodo (2003)
Normalized return distribution

366 stocks on the Tokyo stock exchange
How can we explain the stylized facts?

Some of the stylized facts are explained by the mixture of distributions hypothesis (MDH) (Clark(1976))

Returns are described by Gaussian with **time-varying volatility**

\[ R_t = \sigma_t \varepsilon_t \quad \varepsilon_t \sim N(0,1) \]

- Absence of autocorrelations
- Slow decay of autocorrelation in absolute returns
- Fat-tailed (heavy tail) distributions

\( \sigma_t \) ( # of information )

Volume, # of transactions
We assume \[ r_t = \sigma_t \varepsilon_t \quad E[r_t] = 0 \]

Absence of autocorrelations

\[
E[r_t r_{t+\Delta}] = E[\sigma_t \varepsilon_t \sigma_{t+\Delta} \varepsilon_{t+\Delta}] = E[\sigma_t \sigma_{t+\Delta}] E[\varepsilon_t \varepsilon_{t+\Delta}] = 0
\]

Non-vanishing autocorrelation in absolute returns

\[
E[(|r_t| - c)(|r_{t+\Delta}| - c)] = E[\sigma_t \sigma_{t+\Delta}] E[|\varepsilon_t| |\varepsilon_{t+\Delta}|] - c^2 \neq 0 \quad c = E[|r_t|]
\]

Not independent

Slow decay (long autocorrelation time)

\[
E[\sigma_t \sigma_{t+\Delta}]
\]
Fat-tailed distributions

Unconditional return distribution

\[ P(r) = \int_{0}^{\infty} P(\sigma_t^2)P(r \mid \sigma_t^2) d\sigma_t^2 \]

\[ R_t = \sigma_t \epsilon_t \]

Volatility distribution

\[ h_t = \sigma_t^2 \]

\[ P(h_t) \propto \frac{1}{h_t} e^{-\frac{(\ln h_t - \theta)^2}{2\alpha^2}} \]

\[ P(h_t) \propto \theta^\alpha h_t^{-\alpha-1} e^{-\theta/h_t} \]

We do not know the form of volatility distributions

- Lognormal distribution
  Clark(1973)

- Inverse gamma distribution
  Paretz(1972)

Student-t distribution
Standardized returns will be Gaussian variables with mean 0 and variance 1
\[ \frac{R_t}{\sigma_t} = \varepsilon_t \quad \varepsilon_t \sim \mathcal{N}(0,1) \]

Volatility is unobserved in the markets.

Volatility is estimated by using high-frequency data.

\[ \frac{R_t}{RV_t^{1/2}} = \varepsilon_t \]

Variance = 1  Kurtosis = 3

- Consistency check of the MDH

- Normality of standardized returns
  
  Andersen et al. (2000) : Exchange Rate Returns
  Fleming & Paye (2011) : Stock Returns etc

Here we analyze stock returns on the TSE
Realized Volatility

Andersen, Bollerslev (1998)

Let us assume that the logarithmic price process follows a stochastic diffusion as

\[ d \ln p(t) = \sigma(t) dW(t) \]

\[ \sigma^2_T(t) = \int_{t-T}^{t} \sigma^2(s) ds \]

Integrated volatility (IV) for T period

Realized volatility is defined by a sum of squared finely sampled returns.

\[ RV_t = \sum_{i=1}^{N} r_{t-T+i*k}^2 \]

\[ r_i = \ln p(i) - \ln p(i-k) \]

return calculated using high-frequency data
Let us consider daily volatility.

Usually stock exchange markets are not open for a whole day.

How to deal with the intraday returns during the breaks?
Hansen and Lunde (2005) introduced an adjustment factor $\lambda$ for realized volatility (RV) without including returns in the breaks.

Correct RV so that the average of RV matches the variance of the daily returns.

\[ \sum_{t=1}^{T} (R_t - \bar{R})^2 \]

\[ c = \frac{\sum_{t=1}^{T} RV_t^0}{\sum_{t=1}^{T} RV_t^0} \]

\[ RV_t = cRV_t^0 \]

For standardized returns, this changes the value of variance but not kurtosis.
In order to avoid non-trading hours issue we calculate RV in the two trading sessions separately.

**Morning return**

\[ R_{MS,t} = \ln P_{MS,t}^{Open} - \ln P_{MS,t}^{Close} \]

\[ \frac{R_{MS,t}}{RV_{MS,t}^{1/2}} = \varepsilon_t \]

**Afternoon return**

\[ R_{AS,t} = \ln P_{AS,t}^{Open} - \ln P_{AS,t}^{Close} \]

\[ \frac{R_{AS,t}}{RV_{AS,t}^{1/2}} = \varepsilon_t \]
3. Morning session + Afternoon session

\[ R_{Intra,t} = \ln P_{MS,t}^{Open} - \ln P_{AS,t}^{Close} \]

\[ R_{Intra,t} \left( \frac{RV_{MS,t} + RV_{AS,t}}{2} \right)^{1/2} = \varepsilon_t \]

This could be underestimated

Larger variance is expected
Microstructure noise

Price discreteness, bid-ask spreads, etc.

- Observed prices are contaminated by microstructure noise
  
  \[
  \ln P(t) = \ln P(t) + \xi(t) \\
  \xi(t) : WN (0, \omega^2)
  \]

- Observed returns are also contaminated by noise

  \[
  \bar{r}(t) = r(t) + \eta(t) \\
  \eta(t) = \xi(t) - \xi(t - \Delta t)
  \]

In the presence of noise RV is calculated as follows

  \[
  \overline{RV} = \sum \bar{r}^2 = \sum_{i=1}^{N} \left( r_i + \eta_i \right)^2 = \sum_{i=1}^{N} r_i^2 + 2 \sum_{i=1}^{N} r_i \eta_i + \sum_{i=1}^{N} \eta_i^2
  \]

Noise terms

\[
2N\omega^2
\]
\[
\overline{RV} = RV + 2N\omega^2 = RV \left(1 + \frac{2N\omega^2}{RV}\right)
\]

\[
N = \frac{T}{\Delta t} \quad \text{Sampling frequency (period)}
\]

\[
\overline{RV} = RV \left(1 + \frac{2\omega^2}{RV} \frac{T}{\Delta t}\right)
\]

- When \( N \) is large, the contribution of the noise terms becomes large.

- 5-min frequency is often used for RV construction.
Empirical Results

Our analysis is based on 5 stocks on the Tokyo Stock Exchange
1: Mitsubishi Co.
2: Nomura Holdings Inc.
3: Nippon Steel Co.
4: Panasonic Co.
5: Sony Co.

Volatility signature plot for Mitsubishi Co.

\[ RV(t) = RV \left(1 + \frac{A}{t}\right) \]

Sampling frequency

14% bias at 5min

32% bias at 5min
\[ \frac{\sigma^2}{1 + \frac{a}{dt}} \]

Variance of standardized returns

Noise contribution

Afternoon session

Morning session

Sampling frequency
Kurtosis of standardized returns

Mitsubishi Co.

Rapid increase

Linear decrease

Morning session

Afternoon session

Sampling frequency

0 5 10 15 20 25 30

min
Finite-sample effect

Peters and De Vilder (2006)

Standardized return distribution

\[
P(y) = \frac{\Gamma(k/2)}{\sqrt{\pi k} \Gamma((k-1)/2)} \left(1 - \frac{y^2}{k}\right)^{(k-3)/2}
\]

Moments

\[
E(y^{2m}) = k^m \left(\frac{m-1}{2}\right) \ldots \frac{1}{2} / \left\{ \left(\frac{1}{2} k + m - 1\right) \ldots \frac{1}{2} k \right\}
\]

Variance

\[
E(y^2) = 1
\]

Kurtosis

\[
E(y^4) = \frac{3k}{k+2} = 3 \left(1 - \frac{2}{k+2}\right)
\]

\( k = \# \text{ of samples} \)
Standardized return distribution (theory)

# of samples

- p(x,4.)
- p(x,6.)
- p(x,8.)
- p(x,12.)
- p(x,24.)
- gauss(x,y)
Mitsubishi Co.

Fitting Results

\[ A_0 \left(1 - \frac{2}{A_1 / dt + 2}\right) \]

Morning session

Afternoon session
Afternoon session

Morning session

\[
A_0 \left(1 - \frac{2}{A_1/dt + 2}\right)
\]
$$\frac{\sigma^2}{1 + \alpha / dt}$$

Variance

dt

- MS+AS
- Afternoon Session
- Morning Session
### Fitting results

#### Variance

<table>
<thead>
<tr>
<th></th>
<th>Mitsubishi</th>
<th>Nomura</th>
<th>Nippon St.</th>
<th>Panasonic</th>
<th>Sony</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS</td>
<td>1.07</td>
<td>0.995</td>
<td>1.03</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td>AS</td>
<td>0.95</td>
<td>0.872</td>
<td>0.997</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>MS+AS</td>
<td>1.12</td>
<td>1.02</td>
<td>1.03</td>
<td>1.13</td>
<td>1.01</td>
</tr>
</tbody>
</table>

#### Kurtosis

<table>
<thead>
<tr>
<th></th>
<th>Mitsubishi</th>
<th>Nomura</th>
<th>Nippon St.</th>
<th>Panasonic</th>
<th>Sony</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS</td>
<td>2.92</td>
<td>2.75</td>
<td>2.75</td>
<td>2.91</td>
<td>2.72</td>
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<tr>
<td>AS</td>
<td>3.27</td>
<td>3.31</td>
<td>3.28</td>
<td>3.13</td>
<td>3.01</td>
</tr>
<tr>
<td>MS+AS</td>
<td>2.72</td>
<td>2.79</td>
<td>2.95</td>
<td>2.73</td>
<td>2.83</td>
</tr>
</tbody>
</table>
Higher moments

\[ E(y^6) = \frac{5k}{k+4} \frac{3k}{k+2} \Rightarrow 15 \]

\[ E(y^8) = \frac{7k}{k+6} \frac{5k}{k+4} \frac{3k}{k+2} \Rightarrow 105 \]

\[ E(y^{10}) = \frac{9k}{k+8} \frac{7k}{k+6} \frac{5k}{k+4} \frac{3k}{k+2} \Rightarrow 945 \]
\[ f(\Delta t) = \frac{a}{k + 4} \frac{k^2}{k + 2} \]

\[ k = \frac{N}{\Delta t} \]

\[ a = 14.7 \]
$f(\Delta t) = \frac{a}{k+6} \frac{k^3}{k+4} \frac{1}{k+2}$

$k = \frac{N}{\Delta t}$

$a = 102$
$f(\Delta t) = \frac{a}{k + 8} \frac{k^4}{k + 6} \frac{1}{k + 4} \frac{1}{k + 2}$

$k = \frac{N}{\Delta t}$

$a = 895$
Autocorrelation of standardized returns

We assume \( r_t = \sigma_t \varepsilon_t \)

**Autocorrelation of returns is insignificant**

\[
E[r_t r_{t+\Delta}] = E[\sigma_t \varepsilon_t \sigma_{t+\Delta} \varepsilon_{t+\Delta}] = E[\sigma_t \sigma_{t+\Delta}]E[\varepsilon_t \varepsilon_{t+\Delta}]= 0
\]

**Autocorrelation of absolute returns is not necessarily zero**

\[
E[(|r_t| - c)(|r_{t+\Delta}| - c)] = E[\sigma_t \sigma_{t+\Delta}]E[\varepsilon_t \varepsilon_{t+\Delta}]- c^2 \neq 0 \quad c = E[|r_t|]
\]

For standardized returns

\[
\frac{r_t}{\sigma_t} = \varepsilon_t
\]

Autocorrelation is always zero not only for returns but also for absolute returns
Absolute standardized returns show no autocorrelation

Morning session

Mitsubishi Co.

Afternoon session

$|R_t|$  
$|\text{Standardized } R_t|$
Some references

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DOI: 10.1143/PTPS.194.43

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JPS Conference Proceedings 1 (2014) 019007
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Tetsuya Takaishi
Analysis of Realized Volatility in Superstatistics
DOI:10.14441/eier.7.89
Cross correlation matrix

1: daily return

\[ r_i(t) = \ln p_i(t) - \ln p_i(t-1) \]

Stock # \( i = 1, \ldots, 366 \)

Correlation matrix

\[ C_{ij} = \frac{E((r_i(t) - \bar{r}_i)(r_j(t) - \bar{r}_j))}{\sigma_i \sigma_j} \]

\[ \bar{r}_i = E(r_i(t)) \]

2: Absolute daily return

\[ r_i(t) = \left| \ln p_i(t) - \ln p_i(t-1) \right| \]

\[ \overline{C}_{ij} = \frac{E((r_i(t) - \bar{r}_i)(r_j(t) - \bar{r}_j))}{\sigma_i \sigma_j} \]
Dynamical cross correlation

Cross correlations are calculated with a rolling window of 400 days.

\[ C_{ij} = \frac{\langle r_i(t) - \langle r_i(t) \rangle \rangle \langle r_j(t) - \langle r_j(t) \rangle \rangle}{\sigma_i \sigma_j} \]

a rolling window of 400 days
Average of off-diagonal elements

\[ \frac{2}{N(N-1)} \sum_{i>j} C_{ij} \]
Average of negative elements most positively correlated
Next we calculate \textit{eigenvalues} of cross correlation matrix.

Then using eigenvalues we construct the CRF which is a portion of the system variance.

\[
CRF(k) = \frac{\sum_{j=1}^{k} \lambda_j}{\sum_{j=1}^{N} \lambda_j} = \frac{\sum_{j=1}^{k} \lambda_j}{\text{Trace}[C]} = \frac{\sum_{j=1}^{k} \lambda_j}{N}
\]
Cumulative Risk Fraction

\[ CRF (k) = \frac{\sum_{j=1}^{k} \lambda_j}{\sum_{j=1}^{N} \lambda_j} \]
$$CRF\; (k) = \frac{\sum_{j=1}^{k} \lambda_j}{\sum_{j=1}^{N} \lambda_j}$$

Absolute daily return
• Results from daily returns and absolute returns are similar.

• Only the first CRF is enough to see the dynamical behavior of the system.

• It is known that the largest eigenvalue corresponds to a market-wide influence to all stocks.
The September 11 attacks

Financial Crisis due to Bankruptcy of Lehman Brothers

10/8/2008, Nikkei index dropped -9.38%

3/11/2011, Tohoku Region Pacific Coast Earthquake

3/14/2011, Nikkei index -6.18%

3/15/2001, Nikkei index -10.55%

5/23/2013, Nikkei index -7.89%
In order to see the point where the potential risk is high, we define the temporal change of the CRF as

$$\Delta CRF (t) = CRF[t + 1] - CRF[t]$$

Zheng et al. (2012)

This captures the steepest increase point where the risk quickly increases.
\[
\Delta CRF_1(t) = CRF_1(t+1) - CRF_1(t)
\]

The September 11 attacks

Financial Crisis due to Bankruptcy of Lehman Brothers

Tohoku Region Pacific Coast Earthquake

FRB QE3 reduction observation
Financial Crisis due to Bankruptcy of Lehman Brothers

Tohoku Region Pacific Coast Earthquake

FRB QE3 reduction observation

The September 11 attacks
Nikkei Index and CRF

Nikkei Index

ΔCRF Daily Return

A large change may be a possible signal of system fragility.
Comparison with GARCH volatility

Volatility: a measure of market risk, not directly observed in the market

GARCH(m,n) model

Return time series $y_t$

\[ y_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0,1) \]

\[ \sigma_t^2 = \omega + \sum_{i=1}^{m} \alpha_i y_{t-i}^2 + \sum_{j=1}^{n} \beta_j \sigma_{t-j}^2, \]

GARCH(1,1)

\[ \sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2 \]

$\omega, \alpha, \beta$ are determined so that the GARCH model matches return data.

We used the Markov Chain Monte Carlo method for parameter estimation.
Random matrix theory

- Consider random variables
  \[ x_i(t) \quad i : 1, \ldots, N \quad t : 1, \ldots, T \]

Correlation matrix for random variables

\[
G_{ij} = \frac{\langle x_i(t) - \langle x_i(t) \rangle \rangle \langle x_j(t) - \langle x_j(t) \rangle \rangle}{\sigma_i \sigma_j}
\]

Eigenvalue distribution from the random matrix theory

\[
P(\lambda) = \frac{Q}{2\pi} \sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}
\]

for \( Q = \frac{T}{N} \gg 1 \)

Wishart matrix

\[
G = \begin{pmatrix}
1 & G_{12} & \ldots & G_{1N} \\
G_{21} & 1 & \vdots \\
\vdots & \ddots & 1 \\
G_{N1} & \ldots & 1
\end{pmatrix}
\]

\[
\lambda_{\pm} = 1 + \frac{1}{Q} \pm 2 \sqrt{\frac{1}{Q}}
\]
Simulation \( N = 400 \quad T = 1500 \)

\[
P(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda}
\]

\[
Q = \frac{T}{N}
\]

RMT works well!
FIG. 4. $P(\lambda)$ for $C$ constructed from daily returns of 422 stocks for the 7-yr period 1990–1996. The solid curve shows the RMT result $P_{nm}(\lambda)$ of Eq. (6) using $N=422$ and $L=1737$. The dot-dashed curve shows a fit to $P(\lambda)$ using $P_{nm}(\lambda)$ with $\lambda_+$ and $\lambda_-$ as free parameters. We find similar results as found in Fig. 3(a) for 30-min returns. The largest eigenvalue (not shown) has the value $\lambda_{422} = 46.3$. 

$N = 422 \quad T = 1737$

$\Delta t = 1 \text{ day}$

1990–96
$N = 366 \quad T = 400$
Inverse participation ratio

- Consider components of an eigenvector $u_k$

Eigenvalues are normalized
$$\sum_{i=1}^{N} (u_{ki})^2 = 1$$

According to the RMT, the distribution of components of an eigenvector is the normal distribution.

$$P(u_k) = \sqrt{\frac{N}{2\pi}} \exp(-Nu^2 / 2)$$

Inverse participation ratio (IPR)

$$IPR = \sum_{i=1}^{N} (u_{ki})^4 \quad \rightarrow \quad N \int u^4 \sqrt{\frac{N}{2\pi}} \exp(-Nu^2 / 2) du = \frac{3}{N}$$
Three limiting cases:

(1) Delocalized state (RMT)

\[ IPR = N \int u^4 \sqrt{\frac{N}{2\pi}} \exp(-Nu^2/2) du = \frac{3}{N} \]

(2) Delocalized state

\[ u_{k_1} = u_{k_2} = u_{k_3} = \cdots = u_{k_N} = \frac{1}{\sqrt{N}} \quad IPR = \frac{1}{N} \]

(3) Localized state

\[ u_{k_1} = 1, u_{k_2} = u_{k_3} = \cdots = u_{k_N} = 0 \quad IPR = 1 \]
Inverse participation ratio

Simulation result

$$IPR = \sum_{i=1}^{N} (u_{ki})^4$$

$N = 400$  $T = 1500$

RMT
Inverse participation ratio

The difference from RMT may contain some information on the risk.

\[ IPR_k = \sum_{l=1}^{N} u_{kl}^4 \]

\[ N \int u^4 \sqrt{\frac{N}{2\pi}} \exp(-Nu^2/2) du = \frac{3}{N} \]
The first eigenvector components are delocalized.
The first eigenvector

The change of IPR can also capture the points where the risk is high.
Conclusion

- We performed the principal components analysis for 366 stocks traded on the Tokyo Stock Exchange from 05/01/1998 to 30/12/2013.
- We found that three sharp increases in the cumulative risk. The three sharp increases found in the cumulative risk fraction can be identified with “Bankruptcy of Lehman Brothers”, “2011/11/3 Tohoku Region Pacific Coast Earthquake” and “FRB QE3 reduction observation “.
- The three financial incidents can also be detected by the change of cumulative risk fraction. A large change may be a possible signal of system fragility.
- We also calculated the Inverse participation ration (IPR). The first IPR is different from the RMT. The change of IPR shows the similar results with the CRF but the signal is not clearly seen.
Reference

Backup
FIG. 11. (a) Inverse participation ratio (IPR) as a function of eigenvalue $\lambda$ for the random cross-correlation matrix $R$ of Eq. (6) constructed using $N=1000$ mutually uncorrelated time series of length $L=6448$. IPR for $C$ constructed from (b) 6448 records of 30-min returns for 1000 stocks for the 2-yr period 1994–1995, (c) 1737 records of 1-day returns for 422 stocks in the 7-yr period 1990–1996, and (d) 1737 records of 1-day returns for 422 stocks in the 7-yr period 1983–1989. The shaded regions show the RMT bounds $[\lambda_+, \lambda_-]$. 