Realized Volatility Analysis in A Spin Model of Financial Markets

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Introduction

Stylized properties of asset returns

- Absence of autocorrelations
- Slow decay of autocorrelation in absolute returns
- Fat-tailed (heavy tail) distributions
- Volatility clustering
- Leverage effect
- Volume/volatility correlation
- .....
FIG. 3. (a) Semilog plot of the autocorrelation function for the S&P 500 returns $G_{\Delta t}(t)$ sampled at a $\Delta t = 1$ min time scale, $C_{\Delta t}(\tau) = [\langle G_{\Delta t}(t) G_{\Delta t}(t+\tau) \rangle - \langle G_{\Delta t}(t) \rangle^2] / [\langle G_{\Delta t}(t)^2 \rangle - \langle G_{\Delta t}(t) \rangle^2]$. 

Gopikrishnan et al. (1999)
Fat-tailed return distribution

Gopikrishnan et al., cond-mat/9905305
Oct. 19 1987

Gopikrishnan et al., cond-mat/9905305

volatility clustering

price return

S&P 500

Price returns

Time
How can we explain the stylized facts?

Some of the stylized facts are explained by the mixture of distributions hypothesis (MDH) (Clark 1976)

Returns are described by Gaussian with time-varying volatility

\[ R_t = \sigma_t \varepsilon_t \quad \varepsilon_t \sim N(0,1) \]

Consistent

- Absence of autocorrelations
- Slow decay of autocorrelation in absolute returns
- Fat-tailed (heavy tail) distributions

\[ \sigma_t \quad ( \# \ of \ information ) \]

Volume, # of transactions
We assume \( r_t = \sigma_t \varepsilon_t \) \( E[r_t] = 0 \)

Absence of autocorrelations

\[
E[r_t r_{t+\Delta}] = E[\sigma_t \varepsilon_t \sigma_{t+\Delta} \varepsilon_{t+\Delta}] = E[\sigma_t \sigma_{t+\Delta}] E[\varepsilon_t \varepsilon_{t+\Delta}] = 0
\]

Non-vanishing autocorrelation in absolute returns

\[
E[(|r_t| - c)(|r_{t+\Delta}| - c)] = E[\sigma_t \sigma_{t+\Delta}] E[|\varepsilon_t| |\varepsilon_{t+\Delta}|] - c^2 \neq 0 \quad c = E[|r_t|]
\]

Not independent

Slow decay (long autocorrelation time)
Unconditional return distribution

\[
P(r) = \int_{0}^{\infty} P(\sigma_t^2)P(r | \sigma_t^2)d\sigma_t^2
\]

\[
P(r | \sigma_t^2) = (2\pi\sigma_t^2)^{-1/2} \exp(-\frac{r^2}{2\sigma_t^2})
\]

Volatility distribution

\[h_t = \sigma_t^2\]

\[
P(h_t) \propto \frac{1}{h_t} e^{-(\ln h_t - \theta)^2/(2\alpha^2)}
\]

\[
P(h_t) \propto \theta^\alpha h_t^{-\alpha-1} e^{-\theta/h_t}
\]

We do not know the form of volatility distributions

- Lognormal distribution
  Clark(1973)

- Inverse gamma distribution
  Paretz(1972)

Student-t distribution
Consistency check of the MDH

\[ R_t = \sigma_t \varepsilon_t \]

Standardized returns will be Gaussian variables with mean 0 and variance 1

\[ \frac{R_t}{\sigma_t} = \varepsilon_t \]
\[ \varepsilon_t \sim \mathcal{N}(0,1) \]

Volatility is unobserved in the markets.

Volatility is estimated by using high-frequency data.

\[ \frac{R_t}{RV_t^{1/2}} = \varepsilon_t \quad \text{Variance} = 1 \quad \text{Kurtosis} = 3 \]

Normality of standardized returns has been examined

Andersen et al. (2000) : Exchange Rate Returns
Fleming & Paye (2011) : Stock Returns
Spin financial models

Argents are assigned to spins

A dynamics is introduced and spins interact each other

Various models have been proposed

- Cont and Bouchaud (2000)
- Stauffer and Penna (1998)
- Iori (1999)
- Sznajd-Weron and Weron (2002)
- Sanchez (2002)
- Bornholdt (2001)
- Takaishi (2005)
- etc

These models appear to show some of the stylized facts

- Volatility clustering
- Fat-tailed distribution
- etc

In this study we examine the MDH for the return dynamics in a spin financial markets (Bornholdt Model)
Realized Volatility

\[ d \ln p(t) = \sigma(t) dW(t) \]

\[ \sigma_T^2(t) = \int_{t-T}^{t} \sigma^2(s) ds \]

\[ RV_t = \sum_{i=1}^{N} r_{t-T+i*k}^2 \]

\[ r_i = \ln p(i) - \ln p(i-k) \]

Integrated volatility (IV) for T period

Andersen, Bollerslev (1998)

\[ \sigma(t) : \text{spot volatility} \]

W(t): Standard Brownian motion

\[ N = \frac{T}{\Delta t} \]

\[ \Delta t: \text{sampling period} \]

Realized volatility is defined by a sum of squared finely sampled returns.

IV \ (\Delta t \to 0)
Finite-Sample Effect

Peters and De Vilder (2006)

Standardized return distribution

\[
P(y) = \frac{\Gamma\left(\frac{k}{2}\right)}{\sqrt{\pi k} \Gamma\left(\frac{(k - 1)}{2}\right)} \left(1 - \frac{y^2}{k}\right)^{(k-3)/2}
\]

Moments

\[
E(y^{2m}) = k^{2m} (2k - 1) \ldots 1 / \{(k + 2m - 4) \ldots k\}
\]

Variance

\[
E(y^2) = 1
\]

Kurtosis

\[
E(y^4) = \frac{3k}{k + 2} = 3(1 - \frac{2}{k + 2})
\]
Standardized return distribution (stock data)
Spin model

We may assign +1 state to “Buy order” and -1 state to “Sell order”


Agents live at sites on an n-dimensional lattice
(In this study we use 2-dimensional lattice.)

Each site has a spin.

\( S_i \) takes +1 or -1

Buy \rightleftharpoons \text{Sell}
\[ M(t) = \frac{1}{n} \sum_{j} S_j(t) \]  

Difference between buy and sell orders

\[ h_i(t) = \sum_{j=1}^{n} J_{ij} S_j(t) - \alpha S_i(t) |M(t)| \quad \alpha > 0 \]

Local interaction: Majority effect  
Global interaction

Spins are updated by the following probability

\[ S_i(t+1) = +1 \quad p = 1/(1 + \exp(-2\beta h_i(t))) \]
\[ S_i(t+1) = -1 \quad 1 - p \]
Simulation Study

\[ M(t) = \frac{1}{n} \sum_{j} S_j(t) \]

\[ r(t) = \frac{[M(t) - M(t-1)]}{2} \]
L=125   β=1.8   α=22

One sweep (one day) = 125x125 updates
No finite-sample effect
Moments of standardized returns

Fitting function

\[ Ak^{2m} / \left\{ (k + 2m - 4) \ldots k \right\} \]

\( k = N / \Delta t \)

Finite-sample effect is seen
Fitting results

<table>
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<tr>
<th></th>
<th>variance</th>
<th>kurtosis</th>
<th>6th</th>
<th>8th</th>
<th>10th</th>
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<td>theory</td>
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<td>3</td>
<td>15</td>
<td>105</td>
<td>945</td>
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<td>Spin model</td>
<td>1.002(9)</td>
<td>2.96(3)</td>
<td>14.72(4)</td>
<td>102.8(4)</td>
<td>926(5)</td>
</tr>
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</table>

Moments of the standardized returns are very close to the values expected for normal variables

Consistent with the MDH
Conclusion

- Spin financial models can reproduce some stylized facts of financial markets.
- Some stylized facts can be explained by the MDH. To examine the MDH in the spin financial market, we calculated the realized volatility and then calculated moments of the returns standardized by the realized volatility.
- We confirmed that the moments of the standardized returns receive the finite-sample effect. However the values in the limit of $\Delta t=1$ are very close to the theoretical values for normal variables. Therefore it is concluded that the return dynamics of the spin financial market is consistent with the MDH.
References