Analysis of Realized Volatility in Superstatistics

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Outline

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- Realized Volatility
- Superstatistics
- Stock Data & Results
- Conclusions

Introduction

In finance volatility is an important value for option pricing, portfolio selection, risk management, etc.

Price return $r_t = \ln p(t) - \ln p(t - \Delta t)$

$$r_t = \sigma_t \varepsilon_t \qquad \varepsilon_t \sim \mathrm{N}(0,1)$$

volatility

However volatility is not a direct observable from asset prices.

We need to estimate volatility by a certain method.

Model estimation of volatility

Make a model to mimic volatility properties

- Volatility clustering
- Fat-tailed distribution

Stylized facts of financial prices

ARCH model	Engle(1982)
GARCH model	Bollerslev(1986)
QGARCH model	Engle, Ng(1993), Sentana(1995)
EGARCH model	Nelson(1991)
GJR-GARCH model	Glosten, Jagannathan, Runkle(1993)
etc.	

GARCH(1,1) model

Bollerslev(1986)

$$y_t = \sigma_t \varepsilon_t ,$$

$$\sigma_t^{\ 2} = \omega + \alpha\ y_{t-1}^{\ 2} + \beta\ \sigma_{t-1}^{\ 2}$$

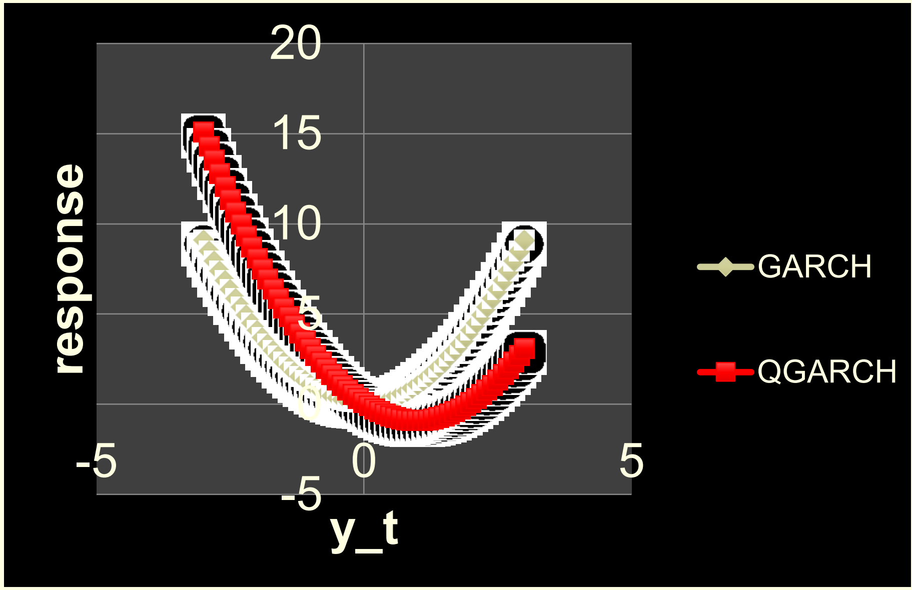
$$\varepsilon_t \sim N(0,1)$$

QGARCH model

Engle,Ng(1993)

Sentana(1995)

$$\sigma_t^{\ 2} = \omega + \gamma y_{t-1} + \alpha\ y_{t-1}^{\ 2} + \beta\ \sigma_{t-1}^{\ 2}$$



Problem in model estimations

The value estimated may depend on the model we use.

Recently, there has been growing interest in using high-frequency data to construct realized volatility.

Realized volatility : a model-free estimate of volatility

1. We measure RV using high-frequency data of some stocks traded on the Tokyo stock exchange and analyze the distributions of RV.

Beck and Cohen (2003) proposed Superstatistics in nonequilibrium system.

Two time scales → superposition of two distributions

2. Here we study RV distributions and try to see whether the price return distribution on the Tokyo stock exchange is considered to be a superposition of two distributions.

Realized Volatility

Andersen, Bollerslev (1998)

A model-free estimate of volatility

Let us assume that the logarithmic price process follows a stochastic diffusion as

$$d \ln p(t) = \mu(t)dt + \sigma(t)dW$$

drift term

daily volatility at day *t*

$$\sigma_t^2 = \int_{t-1}^{t} \sigma^2(s)ds$$

Integrated volatility (IV)

Daily realized volatility is defined by

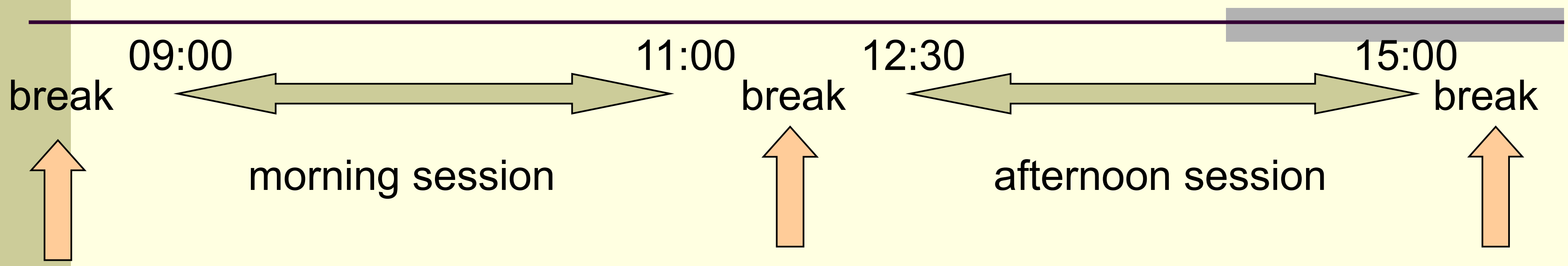
$$r(s) = \ln p(s) - \ln p(s - \Delta s)$$

$$RV_t = \sum_{i=1}^{n} r_{t-1+i/n}^2 \Longrightarrow \text{IV}$$

intraday return calculated using high-frequency data

A problem in calculating RV

Domestic stock trade at the Tokyo stock exchange



How to deal with the intraday returns during the breaks?

Hansen、Lunde(2005)

RV without returns in the breaks → underestimate

R_t :daily return

T: trading days

correct RV so that the average of RV match the variance of the daily returns

$$c = \frac{\sum_{t=1}^{T}(R_t - \overline{R})^2}{\sum_{t=1}^{T} RV_t^0}$$

variance

average

$$RV_t = cRV_t^0$$

Superstatistics

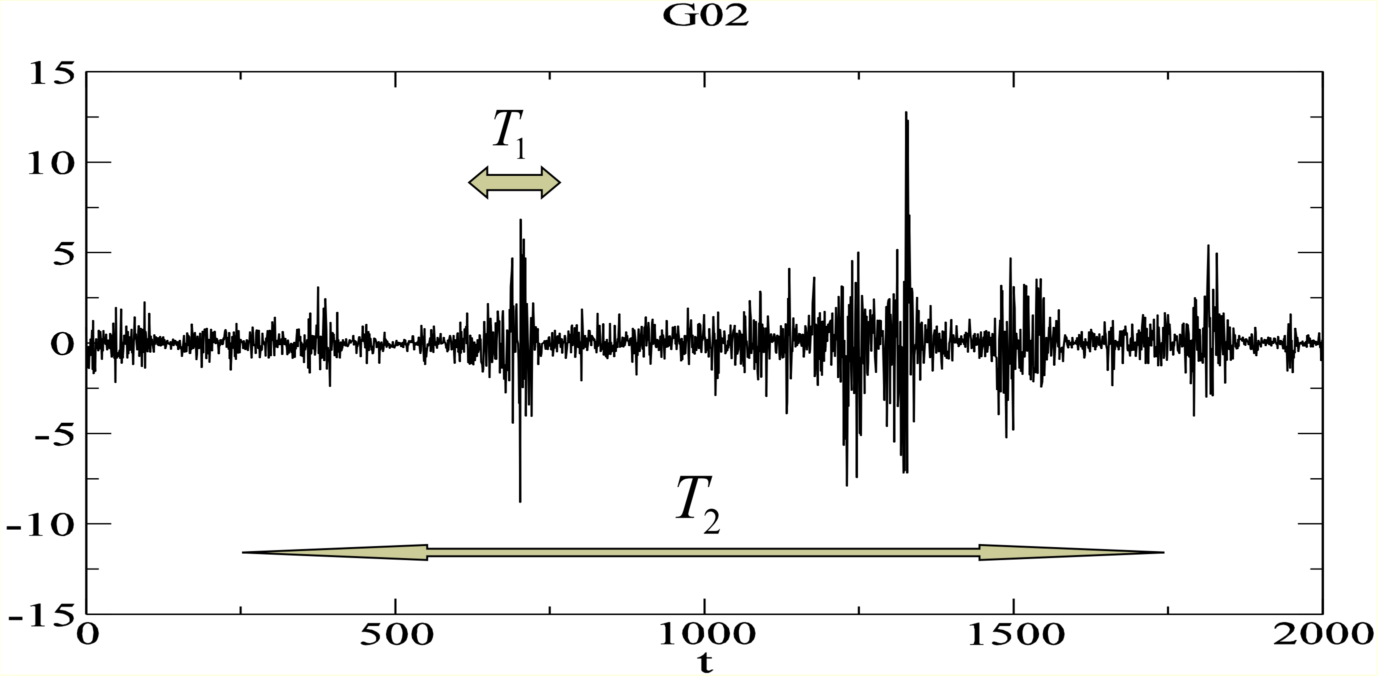
Beck, Cohen (2003)

The daily return distribution is a superposition of two distributions?

Two time scales

•short time scale: equilibrium with a Gaussian distribution with a constant volatility

•long time scale: nonequilibrium with time-varying volatility



Probability distribution of return in a shot time scale

$$P(r \mid \sigma_t^{\ 2}) = (2\pi\sigma_t^2)^{-1/2} \exp(-\frac{r^2}{2\sigma_t^2})$$

Gaussian distribution with a constant volatility

Let us assume that in a long time scale the volatility changes in time with a probability distribution $P(\sigma_t^{\ 2})$

The unconditional probability distribution of return is given as a superposition of two distribution.

$$P(r) = \int_0^\infty P(\sigma_t^2)(2\pi\sigma_t^2)^{-1/2} \exp(-\frac{r^2}{2\sigma_t^2}) d\sigma_t^2$$

$$h_t = \sigma_t^{\ 2}$$

$$P(h_t) \propto \theta^{-\alpha} h_t^{\ \alpha-1} e^{-h_t/\theta}$$

Gamma distribution

$$P(h_t) \propto \frac{1}{h_t} e^{-(\ln h_t - \theta)^2/(2\alpha^2)}$$

Lognormal distribution

$$P(h_t) \propto \theta^{\alpha} h_t^{\ -\alpha-1} e^{-\theta/h_t}$$

Inverse gamma distribution

Stock Data & Results

7 stocks on the Tokyo stock exchange

from March 1, 2006 to February 28, 2008 (493 trading days)

1:Nippon Steel

2:Toyota Motor

3:Sony

4:Nomura Holdings

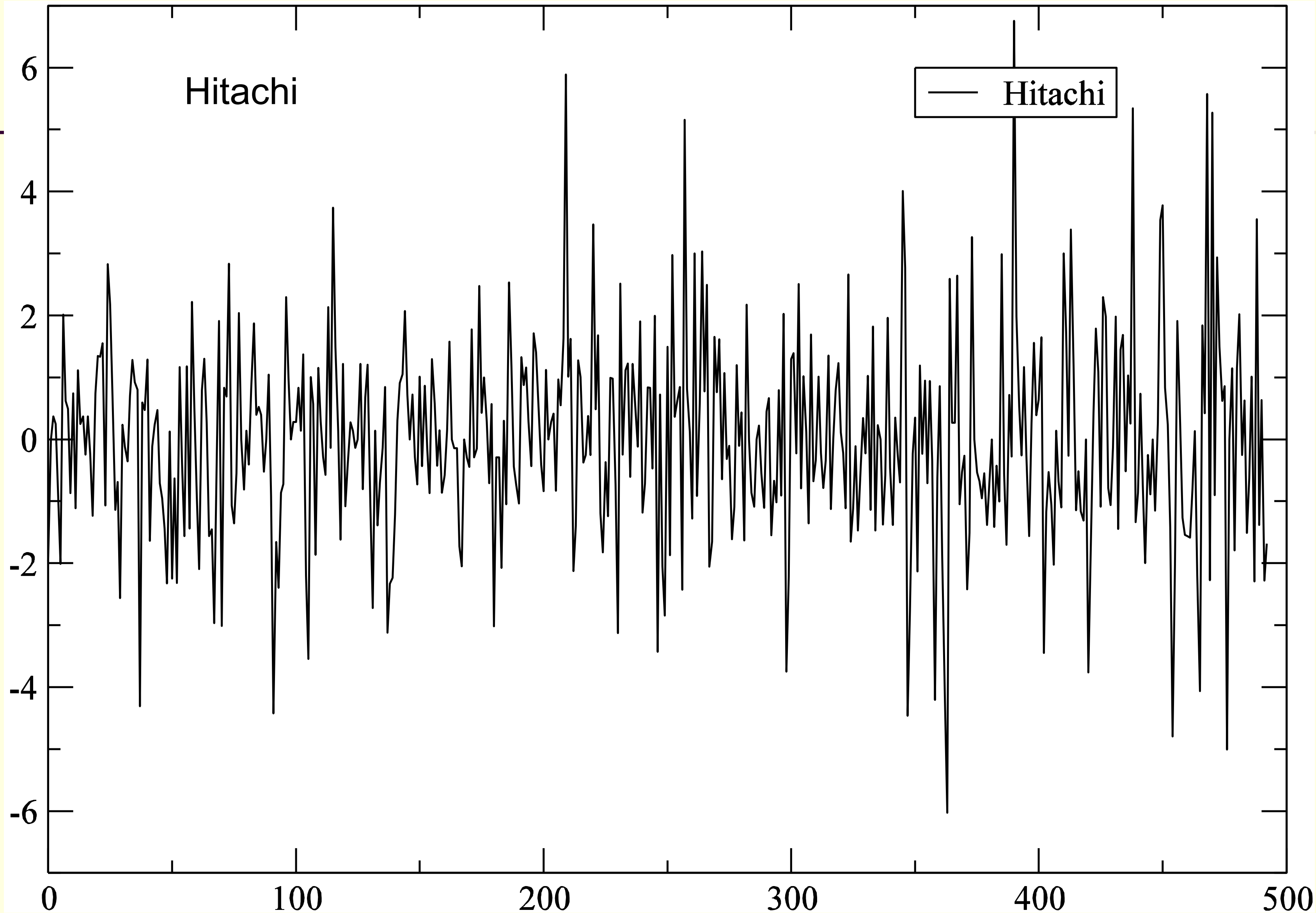
5:Hitachi

6:Daiwa Securities

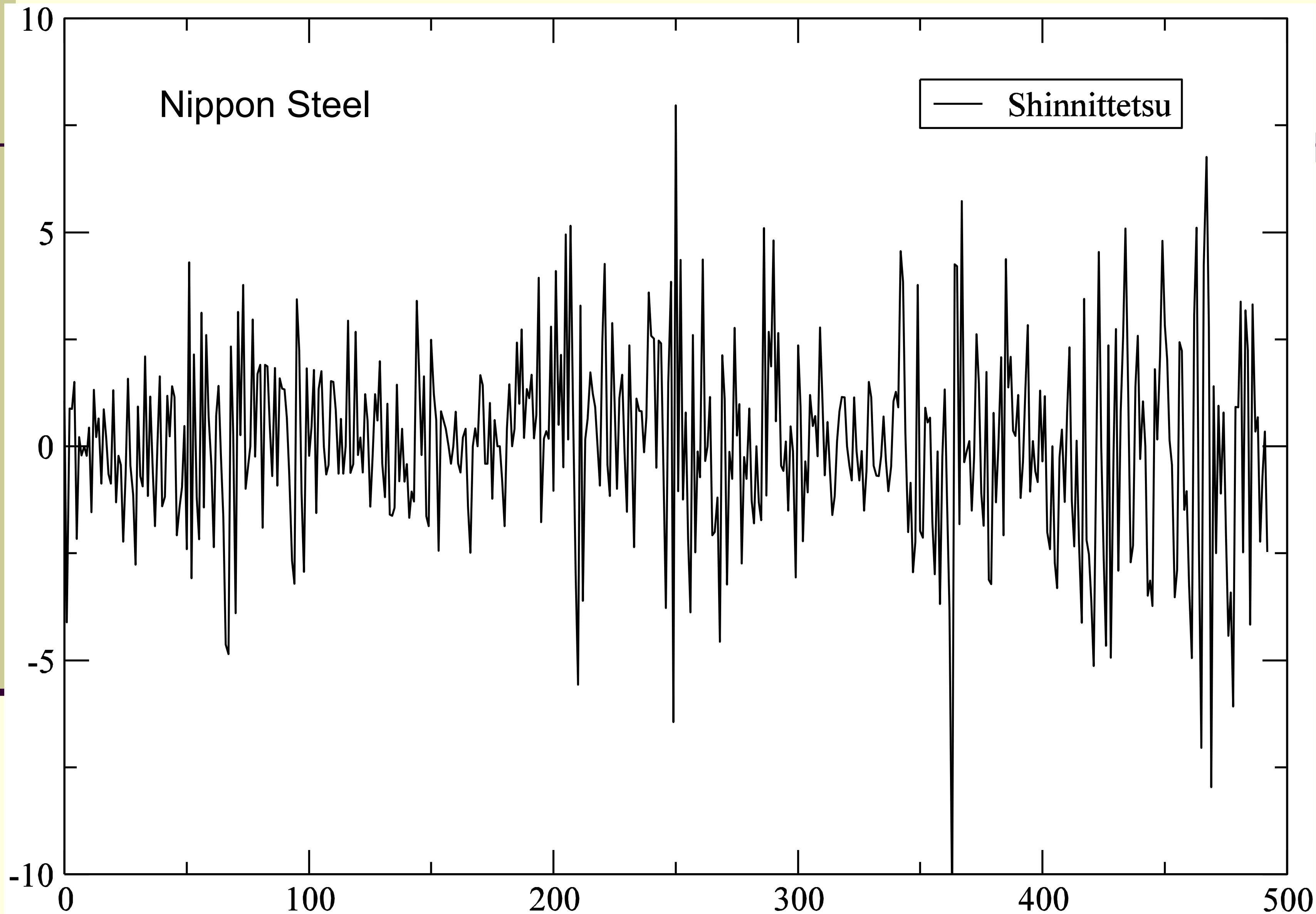
7:Mizuho Financial Group

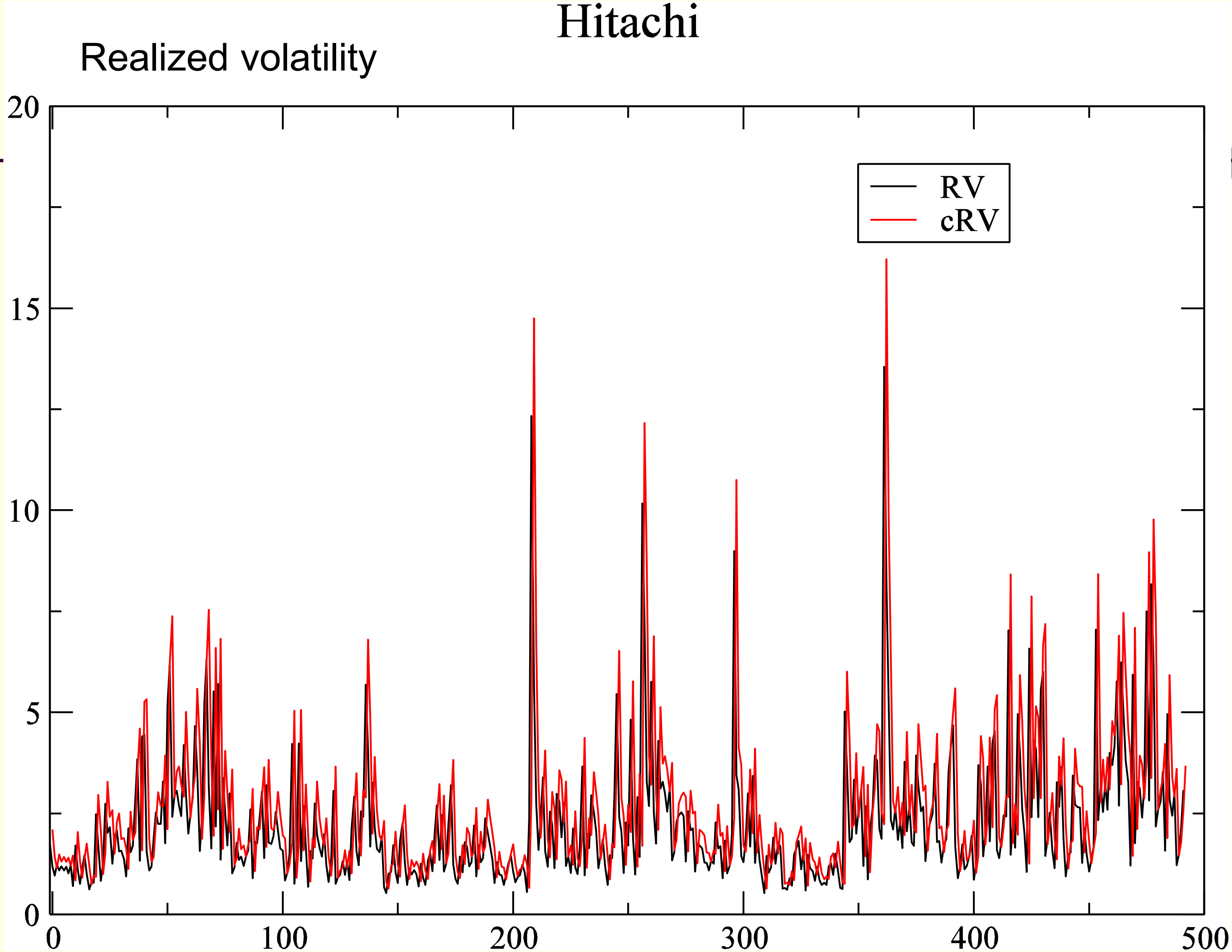
Each realized volatility is calculated using 5-min intraday returns.

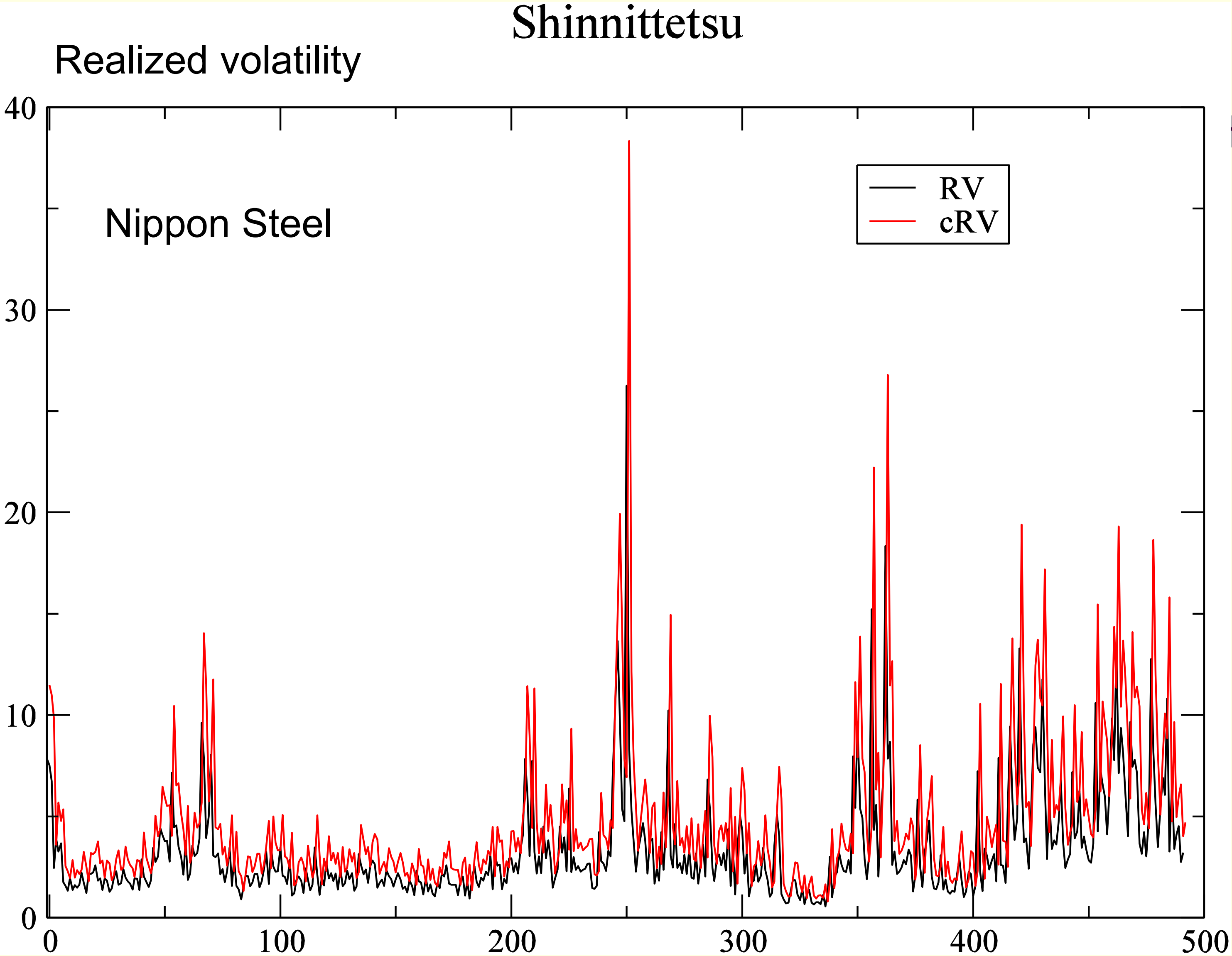
Daily return



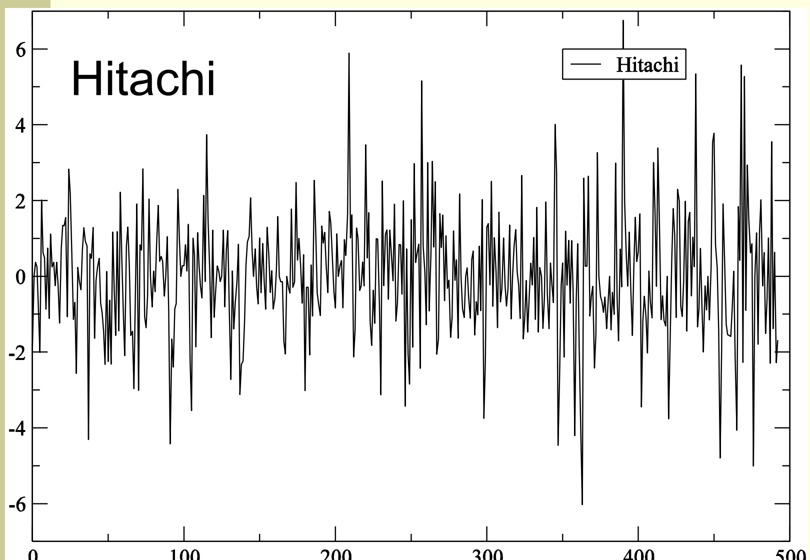
Daily return



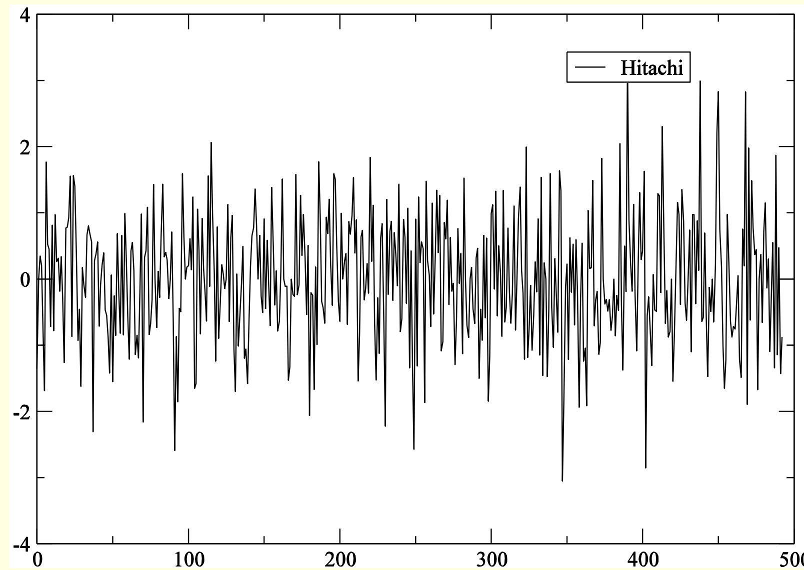

Hitachi
Realized volatility
RV
cRV
0
5
10
15
20
0
100
200
300
400
500


Shinnittetsu
Realized volatility
Nippon Steel
RV
cRV
40
30
20
10
0
0
100
200
300
400
500

$$r(t) = \sigma_t \varepsilon_t \quad \Rightarrow \quad \frac{r(t)}{\sigma_t} = \varepsilon_t \quad \Rightarrow \quad \text{Gaussian}$$

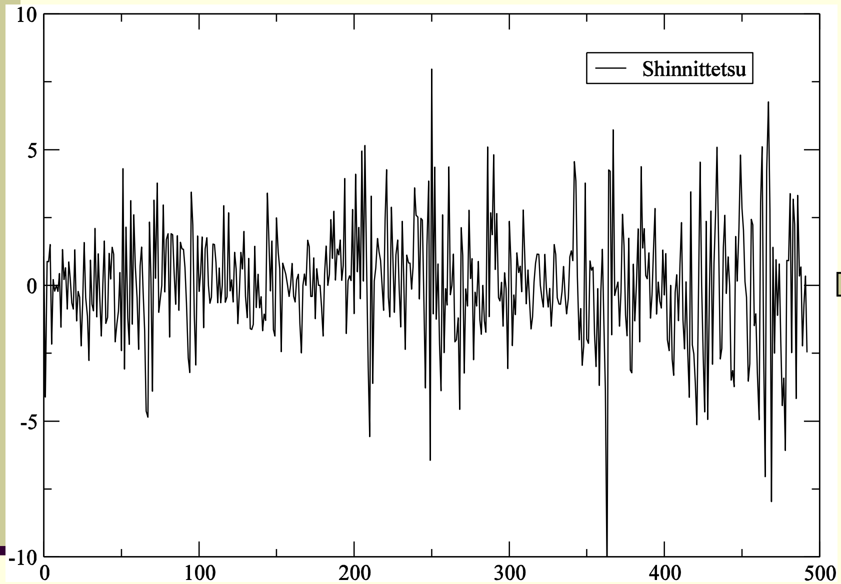


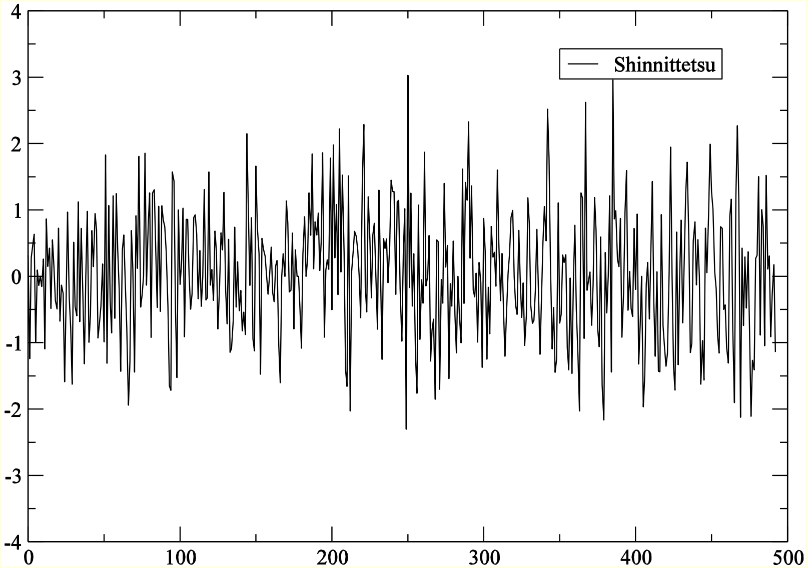
$r(t)$



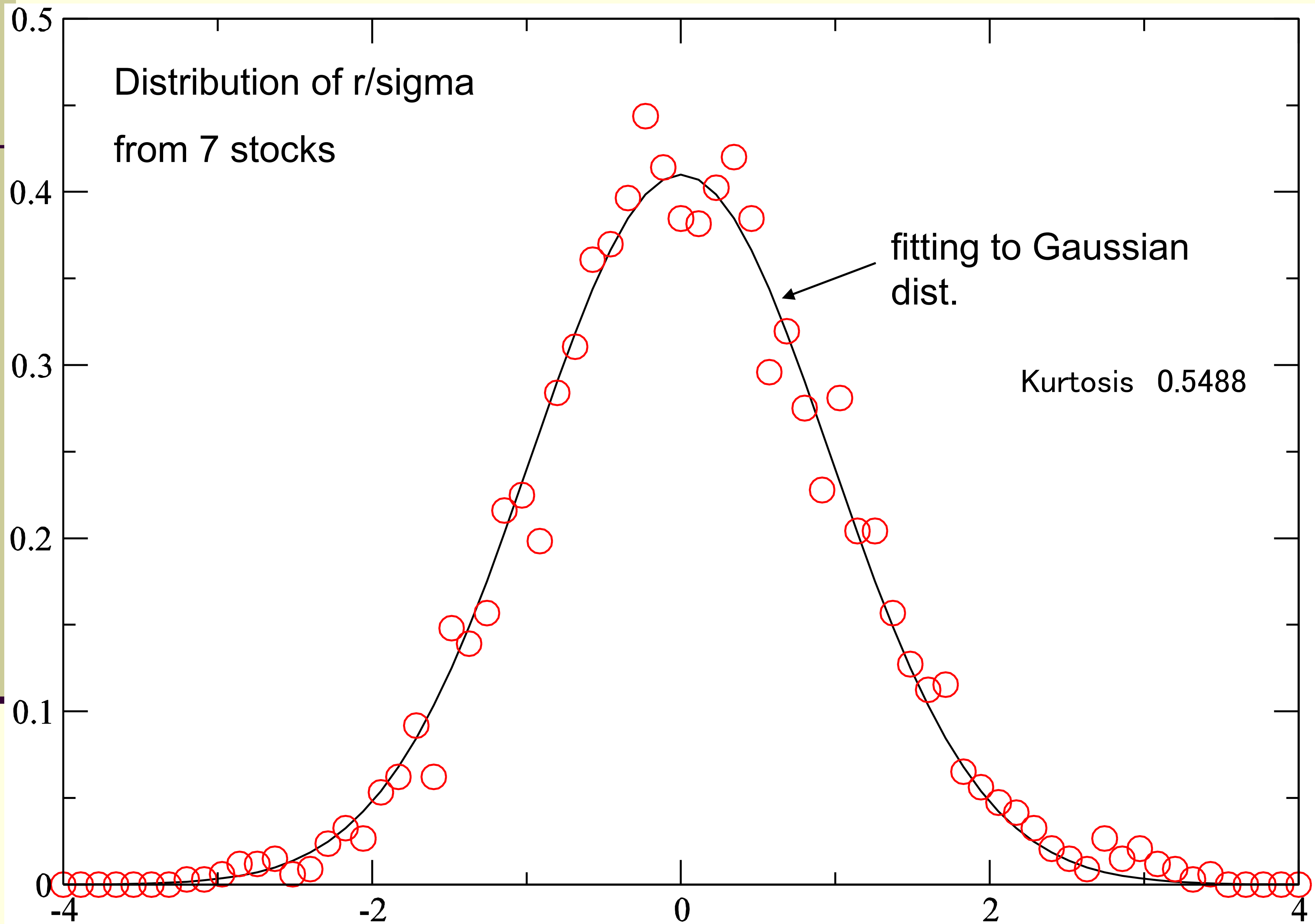
$$\frac{r(t)}{\sigma_t} = \frac{r(t)}{(cRV)^{1/2}}$$

Nippon Steel





		Nippon Steel	Toyota	Sony	Nomura	Hitachi	Daiwa	Mizuho
$r(t)$	var.	4.832	2.588	3.977	4.8479	2.6703	5.7558	2.192
	kurt.	1.6324	2.369	2.072	0.4815	1.7429	0.9704	4.809
$\frac{r(t)}{\sigma_t}$	var.	0.916	0.922	0.990	1.1355	0.9348	1.0459	1.051
	kurt.	-0.180	0.405	0.405	0.7408	0.2120	0.0350	0.548



Distribution of r/sigma
from 7 stocks
fitting to Gaussian dist.
Kurtosis 0.5488
0.5
0.4
0.3
0.2
0.1
0
-4
-2
0
2
4

Distribution of RV

What is the functional form of the distribution of RV?

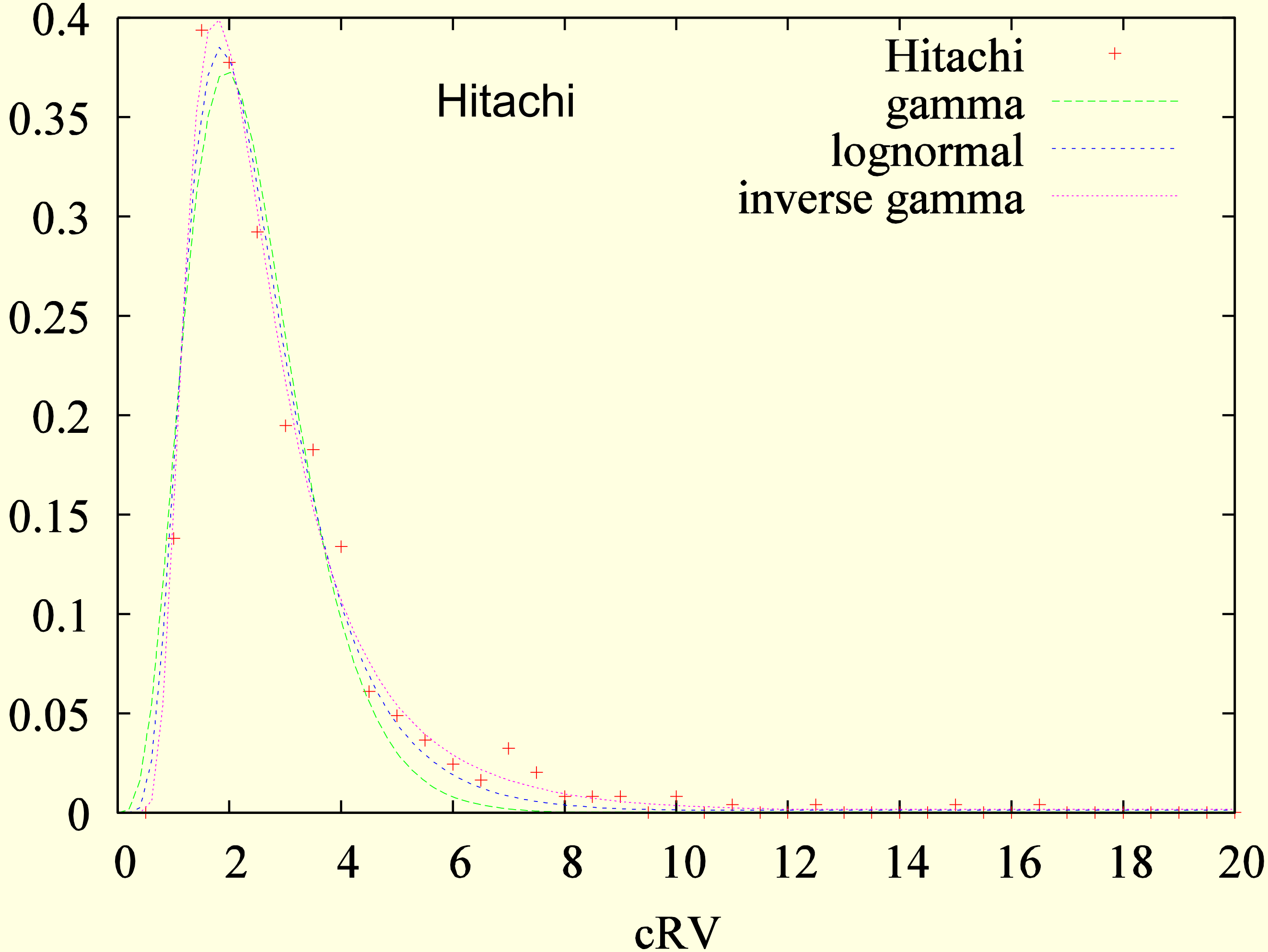
Previously, lognormal or inverse gamma distributions are suggested.

Andersen et al.(2001) : lognormal distribution

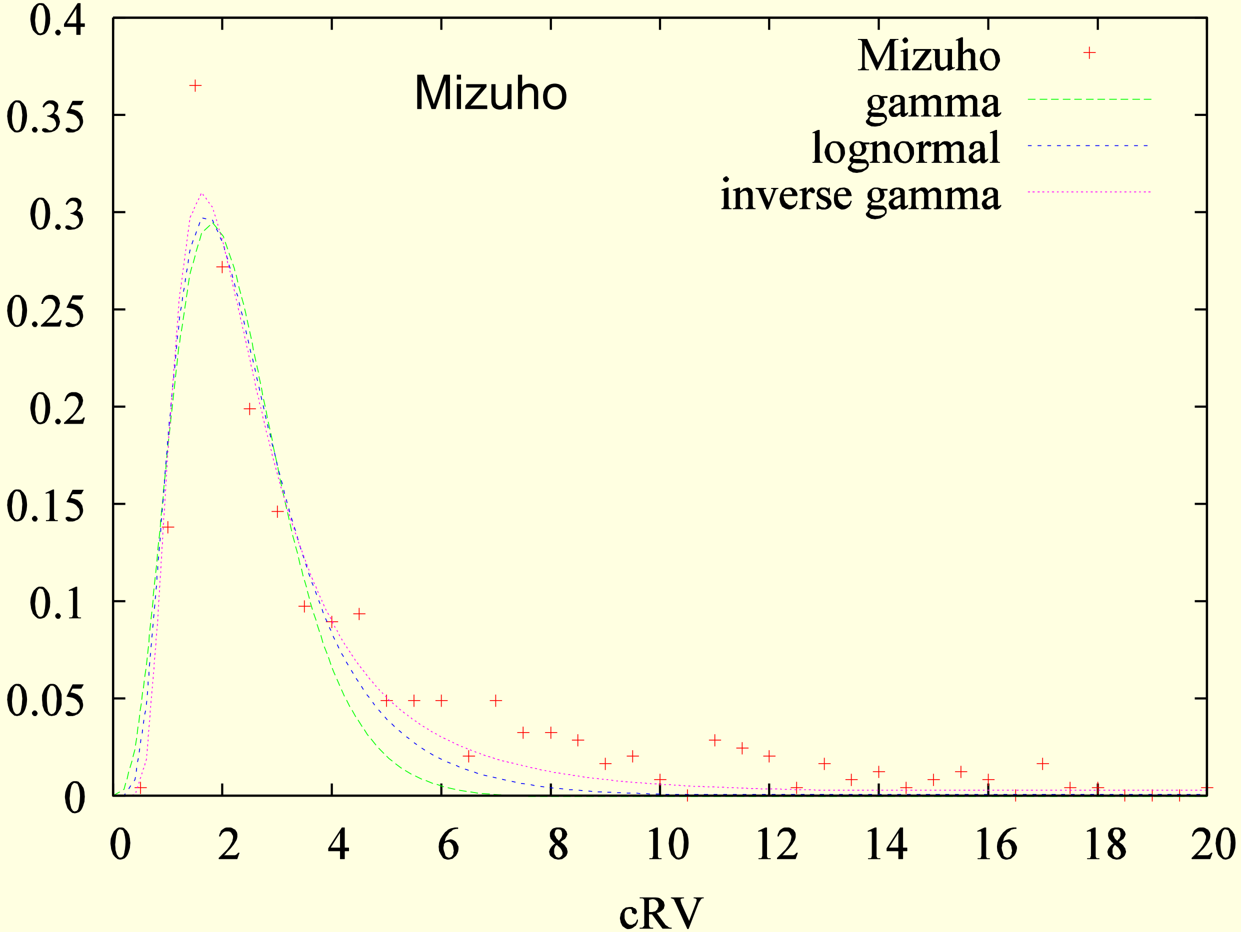
Straeten and Beck(2009): lognormal or inverse gamma

Gerig et al.(2009): inverse gamma

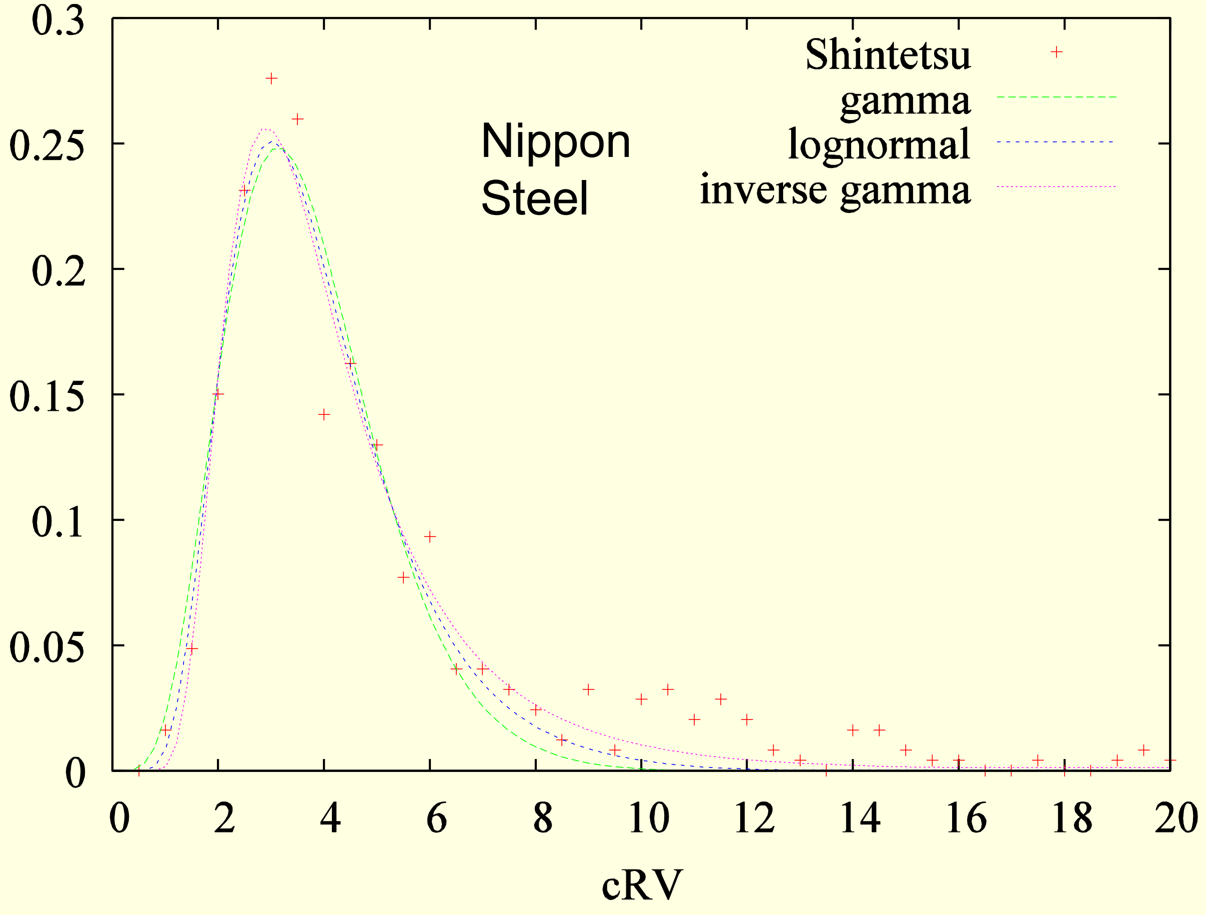
Distribution of RV



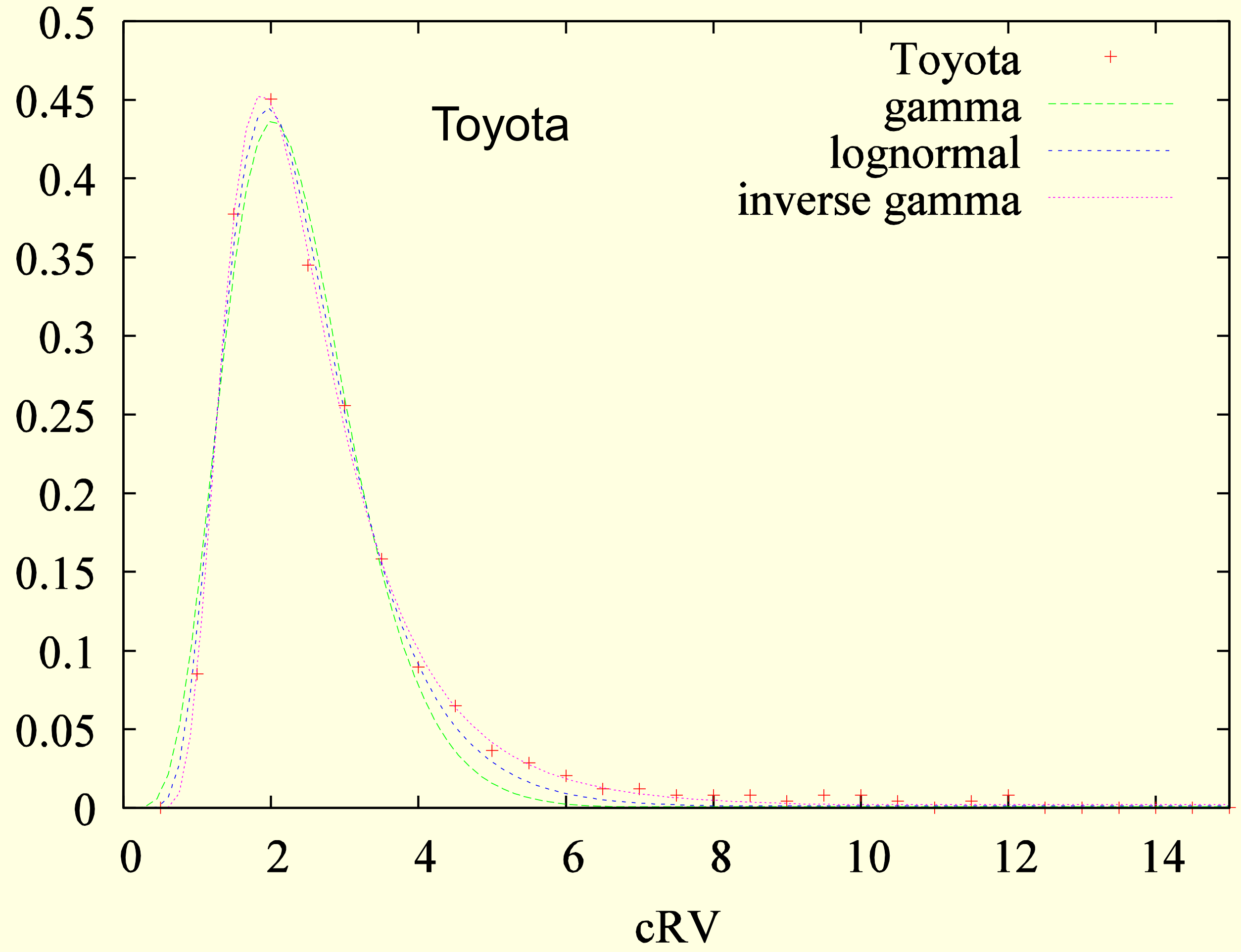
Distribution of RV



Distribution of RV



Distribution of RV



RMS of residuals

	Hitachi	Nippon Steel	Mizuho	Toyota
Gamma	0.0201	0.019	0.028	0.017
Lognormal	0.014	0.0167	0.023	0.00997
IGamma	0.0098	00147	0.018	0.00493

IGamma>Lognormal>Gamma

Conclusions

- We calculated RV for 7 stocks traded in the Tokyo stock exchange market.
- The distribution of the daily return normalized by RV is close to a Gaussian distribution.
- The best fit to RV is given by the inverse gamma distribution.
- Superstatistical view may be possible with a superposition of the inverse gamma and Gaussian distributions.
- But more studies using other stocks are needed to the conclusive answer.

$$f(h_t) \propto \theta^{-\alpha} h_t^{\alpha-1} e^{-h_t/\theta}$$

$$f(h_t) \propto \frac{1}{h_t} e^{-(\ln h_t - \theta)^2/(2\alpha^2)}$$

$$f(h_t) \propto \theta^{\alpha} h_t^{-\alpha-1} e^{-h_t/\theta}$$

Introduction

株価変動P(t)のダイナミクス

幾何ブラウン運動と仮定すると収益率r(t)=ln(P(t)/P(t-1)の変動は正規分布で表される

$$r(t) = \sigma \varepsilon_t$$

実際の市場では

- Volatility Clustering
- Fat-tailed Distribution

正規分布ではない

ボラティリティは時間変動する $r(t) = \sigma_t \varepsilon_t$

$P(\sigma_t^2)$ ボラティリティはある確率分布に従う(Superstatistics)

日本市場の株価において、Realized volatility からボラティリティを見積もり、Superstatistics的な考えが良いかどうかをみる

Realized volatility

実現ボラティリティ

収益率の変動が以下のように表されるとき、ボラティリティをどのように推定するか？

$$r_t = \sigma_t \varepsilon_t$$

収益率　　　ボラティリティ

ボラティリティは直接観測できる量ではないので見積もる必要がある

モデルボラティリティ

例えば、

GARCHモデル、EGARCHモデル、
QGARCHモデル、GJRモデルなど

Realized volatility を測定するときの問題点

マイクロストラクチャーノイズ

$$\bar{r}(t) = r(t) + \eta(t) \qquad \eta(t) : WN(0, \sigma_\eta^2)$$

観測値　真の値　ノイズ

Δ:時間間隔

$$Var(\bar{r}(t) - \bar{r}(t-\Delta)) = Var(r(t) - r(t-\Delta)) + Var(\eta(t) - \eta(t-\Delta))$$

$$Var(r(t) - r(t-\Delta)) = \int_{t-\Delta}^{t} \sigma^2(s)ds \xrightarrow{\Delta \to 0} 0$$

$$Var(\eta(t) - \eta(t-\Delta)) = 2\sigma_\eta^2 \longrightarrow \text{有限}$$

ここでは、5分後とに計算する