Some properties of Multiple Time Series Ising Model in Financial Market Simulations

Tetsuya Takaishi
Hiroshima University of Economics
Introduction

\[ \text{Return} = \ln P(t + 1) - \ln P(t) \]

\[ \text{Nikkei Index} \quad P(t) \]

Volatility clustering
Asset returns are not Gaussian time series.

Statistical properties of asset returns have been intensively studied.

**Stylized facts of asset returns**

- Absence of autocorrelations
- Slow decay of autocorrelation in absolute returns
- Fat-tailed (heavy tail) distributions
- Volatility clustering
- Leverage effect
- Volume/volatility correlation
- .....
FIG. 3. (a) Semilog plot of the autocorrelation function for the S&P 500 returns $G_{\Delta t}(t)$ sampled at a $\Delta t = 1$ min time scale, $C_{\Delta t}(\tau) = \frac{\langle (G_{\Delta t}(t) G_{\Delta t}(t+\tau)) - \langle G_{\Delta t}(t) \rangle^2 \rangle}{\langle (G_{\Delta t}(t))^2 \rangle - \langle (G_{\Delta t}(t))^2 \rangle}$. 

Gopikrishnan et. al (1999)
Fat-tailed return distribution

Gopikrishnan et al., cond-mat/9905305
Oct. 19 1987

volatility clustering

price return

Gopikrishnan et al., cond-mat/9905305
To better understand the dynamics of asset returns we would like to model return time series.

What is a minimalistic model that captures stylized facts that are observed in the real financial markets?

A spin model is introduced to simulate a financial market by Bornholdt.

The model successfully captures several stylized facts such as the volatility clustering.

However only one asset (stock) is considered in the model.

The real financial market consists of many stocks interacting each other.

In this study we would like to extend the one asset model to a model including multiple stocks and investigate properties of the model.
One asset spin model

Two states spin model


Agents live at sites on an n-dimensional lattice.

(In this study we use 2-dimensional lattice.)

Each site has a spin.

\[ S_i \text{ takes } +1 \text{ or } -1 \]

Buy

Sell

We may assign +1 state to “Buy order” and -1 state to “Sell order.”
Magnetization

\[ M(t) = \frac{1}{n} \sum_{j} S_j(t) \]

\[ h_i(t) = \sum_{j=1}^{n} J_{ij} S_j(t) - \alpha S_i(t) |M(t)| \]

Local interaction: Majority effect

Global interaction: Minority effect

Spins are updated by the following probability

\[ S_i(t+1) = +1 \quad p = 1/(1 + \exp(-2\beta h_i(t))) \]

\[ S_i(t+1) = -1 \quad 1 - p \]
Simulation results

\[ M(t) = \frac{1}{n} \sum_{j} S_j(t) \]

\[ r(t) = \frac{[M(t) - M(t-1)]}{2} \]
Return distributions

Cumulative return distributions

Fat-tailed distribution
Real financial markets are complex system that consists of many stocks interacting each other.

Next we extend the one asset model to a model that can simulates multiple stocks.
We introduce K n-dimensional lattices. Each lattice corresponds to one stock.

\[ h_i^{(k)}(t) = \sum_{j=1}^{n} J_{ij} S_j^{(k)}(t) - \alpha^{(k)} S_i^{(k)}(t) |M^{(k)}(t)| + \sum_{j=1}^{K} \gamma_{jl} M^{(j)}(t) \]

\( \gamma_{jl} \) Coupling constant

Interaction strength between stocks
Simulation study

K=3, three stocks

120x120 lattices

Simulation parameters

\((\beta, \alpha, J) = (2.0, 35, 1)\)

Interaction matrix

\[
\gamma = \begin{pmatrix}
0 & 0.05 & 0.1 \\
0.05 & 0 & 0.15 \\
0.1 & 0.15 & 0
\end{pmatrix}
\]
\[ M(t) = \frac{1}{n} \sum_j S_j(t) \]

magnetization
Return

\[ r(t) = \left[ M(t) - M(t-1) \right] / 2 \]
Return distributions

Fat tail

- Stock 1
- Stock 2
- Stock 3
Autocorrelation of return

No long-range correlations are seen

Autocorrelation of absolute return

Long-range correlations are seen
Cross correlation matrix

\[
\frac{\left< (R^{(l)}(t) - \left< R^{(l)}(t) \right>) (R^{(m)}(t) - \left< R^{(m)}(t) \right>) \right>} {\sigma^{(l)} \sigma^{(m)}}
\]

Absolute return

<table>
<thead>
<tr>
<th></th>
<th>Stock 1</th>
<th>Stock 2</th>
<th>Stock 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock 1</td>
<td>1</td>
<td>9.91x10^{-2}</td>
<td>0.133</td>
</tr>
<tr>
<td>Stock 2</td>
<td>9.91x10^{-2}</td>
<td>1</td>
<td>0.212</td>
</tr>
<tr>
<td>Stock 3</td>
<td>0.133</td>
<td>0.212</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\gamma = \begin{pmatrix}
0 & 0.05 & 0.1 \\
0.05 & 0 & 0.15 \\
0.1 & 0.15 & 0
\end{pmatrix}
\]

For returns we do not observe significant cross correlations.
Conclusion

- We extended the one asset model to a more realistic model that can simulates a number of assets.
- We have simulated the multiple time series model with three stocks.
- We found that the multiple time series model reproduces some of major stylized facts observed in the real financial markets: Absence of correlations in return, long correlations in absolute returns, volatility clustering, fat-tailed distributions.
- We confirmed that cross correlations appear between absolute returns. On the other hand no significant correlation is observed between returns.
- It might be interesting to see properties of the model with many stocks.
References