Estimating Bivariate GARCH-Jump Model Based on High Frequency Data *
the case of revaluation of Chinese Yuan in July 2005

Xinhong Lu†, Koichi Maekawa‡, Ken-ichi Kawai §

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Abstract

This paper attempts to model the behavior of 1-minute high frequency exchange rate data of 5 currencies: the Japanese Yen, the Australian Dollar, the Canadian Dollar, the Euro, the Pound sterling against the US Dollar, on 21 July 2005 when the Chinese Yuan was revaluated. The data shows the following distinctive features: (1) There is a large jump at the time of the Yuan revaluation, (2) Large volatility is observed for a while after the jump, (3) There were many other jumps, possibly correlated, in each exchange rate time series. To capture these features we fit the following models to the data: (i) One dimensional GARCH-Jump model with a large jump which is influential on volatility, and (ii) a bivariate GARCH-Jump model with correlated Poisson jumps. For comparison, we also estimate one and two dimensional GARCH model without jumps. The model performance is evaluated based on Value-at-Risk (VaR).

Key Words: High frequency data, bivariate GARCH Jump model, correlated Poisson jumps, VaR threshold

* Bloomberg Data.
† Graduate School of Social Sciences, Hiroshima University.
‡ Faculty of Social Sciences, Hiroshima University.
§ The Institute of Statistical Mathematics.
1 Introduction

We analyze 1-minute high frequency data of time series of exchange rates observed on 21 July 2005 when the Chinese Yuan was revaluated and gave a shock to the time series of exchange rate of the Japanese Yen, the Australian Dollar, Canadian Dollar, Euro, Pound sterling. The data are graphed in Fig.1 - Fig.5. From these we can observe the following distinctive features: The fluctuation of the returns in the high frequency data for the exchange rates exhibits the persistent effect after the Chinese Yuan revaluation. For example, Fig.1 shows the 1-minute intraday data for the Japanese Yen exchange rate against US Dollar and its log-return. The sample period is from 12:00 p.m. on July 21 to 11:59 a.m. July 22. The Chinese Yuan was revalued at 20:00 (8:00 p.m.) on July 21. At the same time, the Japanese Yen appreciated immediately against US Dollar. The volatility fluctuates largely and the effect persists for a while (about 6 hours). Similarly, other exchange rates (Australian Dollar, Canadian Dollar, Euro, Pound sterling) exhibit almost the same feature as the Japanese Yen after the Chinese Yuan revaluation (See Fig.2 - Fig.5). Furthermore jumps in each currency seems correlated. In short the distinctive features are: (1) There is a large jump when the Yuan revaluation was announced, (2) Furthermore large volatility is observed for a while after the jump, (3) There were many other jumps, possibly correlated, in each exchange time series.

To model these observed phenomena we apply two models: one dimensional GARCH-Jump model and a bivariate GARCH model with correlated Poisson jump. We also apply one and two dimensional GARCH models without jumps and compare the performance of these models by using VaR threshold.
This paper is organized as follows: Section 2 describes the data and tests if the 5 exchange rate data have jumps by Bipower Variation (BPV) test (Barndorff-Nielson-Shephard, 2005). Section 3 proposes one dimensional GARCH model to capture the aftereffect of a jump. Section 4 describes a Bivariate GARCH model with correlated Poisson jumps and apply it to the data. Section 5 compares the performance of the models by using VaR threshold. Section 6 offers conclusion.

2 Bipower Variation (BPV) test

First of all, we test if there were jumps in the time series of one minute high frequency of returns of several exchange rates such as the Japanese Yen, the Australian Dollar, the Canadian Dollar, the Euro, the Pound sterling against the US Dollar. Barndorff-Nielsen and Shephard (2005) proposed three formulas of Bipower Variation (BPV) Test, i.e. G-, H-, J- test for testing the null of no jump. We apply BPV J test to the high frequency data in the second half of July 2005 and the test shows that there were jumps in the returns in those exchange rate under 5% critical value -1.28 for J-test (see Table 1).

<table>
<thead>
<tr>
<th>Exchange Rate</th>
<th>J-test Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japanese Yen</td>
<td>J=-3.16</td>
</tr>
<tr>
<td>Euro</td>
<td>J=-2.53</td>
</tr>
<tr>
<td>Australian $</td>
<td>J=-5.38</td>
</tr>
<tr>
<td>Canadian $</td>
<td>J=-4.24</td>
</tr>
<tr>
<td>Pound sterling</td>
<td>J=-4.30</td>
</tr>
</tbody>
</table>

Table 1: Results of BPV J test for exchange rate
3 One dimensional GARCH-Jump model

In this section we proposed a model to capture the phenomenon of the volatility persistence after a jump occurs. We modify the standard GARCH model to take a jump into account. To do so we introduce a constant term in the variance equation which shifts after the jump, and this shockwave exponentially decreases:

\[ y_t = c + \epsilon_t \]
\[ \epsilon_t = \sigma_t \xi_t, \quad \xi_t \sim i.i.d. N(0, 1) \]
\[ \sigma_t^2 = \omega_t + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \]
\[ \omega_t = \begin{cases} \begin{array}{l} a \\ a + b \times exp(\lambda (t^* - t)) \end{array} \end{cases} \]
\[ t = 1, 2, \cdots, T, \]

where \( c, \alpha, \beta \) and \( \lambda \) are parameters, \( t^* \) is the time at which a jump occurs. \( t^* \) is assumed to be known.

For illustration we use the Japanese Yen /US Dollar rate to estimate this model by ML method. The sample size is 1440 and the time point where the revaluation occurred is 480. The estimation result is as follows:

\[ y_t = -0.001420 + \epsilon_t, \ t = 1, \cdots, T \]
\[ \sigma_t^2 = \omega_t + 0.176898 \epsilon_{t-1}^2 + 0.030163 \sigma_{t-1}^2 \]

where
\[ \epsilon_t = \sigma_t \xi_t, \quad \xi_t \sim i.i.d. N(0, 1) \]
\[ \omega_t = \begin{cases} \begin{array}{l} 0.000 \\ 0.000 + exp(0.0000125 \times (t^* - t)^2) \end{array} \end{cases} \]
\[ t = 1, 2, \cdots, 480, \cdots, 1440. \]

(3.1)

Fig. 6 is a sample path simulated by (3.1) by setting \( a = 0.1, b = 1 \) in \( \omega_t \). This figure
suggests that this kind of model could describe a shock and afterwave like fluctuation
in Fig.1 - Fig.5.

4 Bivariate GARCH Model with Correlated Poisson Jumps

Following Chan (2003), we assume the multivariate GARCH model with correlated
jumps to describe our data, that is, we assume that the number of jumps per a unit
time interval follows Poisson distribution. Here we focus on a bivariate case.

Bivariate GARCH model with correlated Poisson jump model is defined as follows:

\[ R_t = \mu + \epsilon_t + \eta_t, \quad t = 1, 2, \ldots, T \]  

(4.1)

where \( R_t \) is a \( 2 \times 1 \) bivariate return vector with a \( 2 \times 1 \) mean vector and two \( 2 \times 1 \)
independent stochastic components vectors \( \epsilon_t \) and \( \eta_t \). \( \epsilon_t \) is a vector of i.i.d. bivariate
normal errors, and \( \eta_t \) is a vector of mean adjusted bivariate Poisson jumps (see eq. (4.8)
below). We can rewrite the model (4.1) in terms of elements of vectors as follows:

\[
\begin{pmatrix}
  r_{1t} \\
  r_{2t}
\end{pmatrix}
= \begin{pmatrix}
  \mu_1 \\
  \mu_2
\end{pmatrix}
+ \begin{pmatrix}
  \epsilon_{1t} \\
  \epsilon_{2t}
\end{pmatrix}
+ \begin{pmatrix}
  \eta_{1t} \\
  \eta_{2t}
\end{pmatrix}
= \begin{pmatrix}
  \mu_1 \\
  \mu_2
\end{pmatrix}
+ \begin{pmatrix}
  u_{1t} \\
  u_{2t}
\end{pmatrix}
\]  

(4.2)

where

\[
\begin{pmatrix}
  u_{1t} \\
  u_{2t}
\end{pmatrix}
= \begin{pmatrix}
  \epsilon_{1t} \\
  \epsilon_{2t}
\end{pmatrix}
+ \begin{pmatrix}
  \eta_{1t} \\
  \eta_{2t}
\end{pmatrix}
\]  

(4.3)

We assume

\[
\begin{pmatrix}
  \epsilon_{1t} \\
  \epsilon_{2t}
\end{pmatrix}
\sim i.i.d.N\left( \begin{pmatrix}
  0 \\
  0
\end{pmatrix}, \begin{pmatrix}
  \sigma^2_{1t} & \sigma^2_{12,t} \\
  \sigma^2_{21,t} & \sigma^2_{2t}
\end{pmatrix}\right)
\]  

(4.4)

Following Engle (1995) we assume that Bivariate GARCH (1,1) structure is written
as:

\[ \epsilon_{1t} = \sigma_{1t} \xi_{1t}, \quad \xi_{1t} \sim i.i.d.N(0,1) \]
\[ \sigma^2_{1t} = \omega_1 + \alpha_1 \epsilon^2_{1,t-1} + \beta_1 \sigma^2_{1,t-1} \]
\[ \epsilon_{2t} = \sigma_{2t} \xi_{2t}, \quad \xi_{2t} \sim i.i.d.N(0,1) \]
\[ \sigma^2_{2t} = \omega_2 + \alpha_2 \epsilon^2_{2,t-1} + \beta_2 \sigma^2_{2,t-1} \]
\[ \sigma_{12,t} = \omega_3 + \alpha_3 \epsilon_{1,t-1} \epsilon_{2,t-1} + \beta_3 \sigma_{12,t-1} \]  

(4.5)
Jump structure is assumed as follows:

During a time period $t$ (from time $t-1$ to $t$) the first currency has $n_{1t}$ jumps and the second one has $n_{2t}$ jumps where

$$n_{1t} \sim \text{Poisson}(\lambda_1) \quad \text{and} \quad n_{2t} \sim \text{Poisson}(\lambda_2)$$

and jump sizes are correlated bivariate normal:

$$Y_{1t,k} \sim N(\theta_1, \delta^2_1) \quad \text{for the first currency} \quad \text{and} \quad Y_{2t,l} \sim N(\theta_2, \delta^2_2) \quad \text{for the second},$$

and

$$\rho_{12} \quad \text{or} \quad (Y_{1t,k} Y_{2t,l}) \sim N\left(\begin{array}{c} \theta_1 \\ \theta_2 \end{array}, \begin{pmatrix} \delta^2_1 & \rho_{12} \delta_1 \delta_2 \\ \rho_{12} \delta_1 \delta_2 & \delta^2_2 \end{pmatrix}\right) \quad \text{(4.6)}$$

and

$$\left(\sum_{k=1}^{n_{1t}} Y_{1t,k} \right) \sim N\left(\begin{array}{c} n_{1t} \theta_1 \\ n_{2t} \theta_2 \end{array}, \begin{pmatrix} n_{1t} \delta^2_1 & \rho_{12} \sqrt{n_{1t} n_{2t}} \delta_1 \delta_2 \\ \rho_{12} \sqrt{n_{1t} n_{2t}} \delta_1 \delta_2 & n_{2t} \delta^2_2 \end{pmatrix}\right) \quad \text{(4.7)}$$

Jump components are defined by

$$\eta_{1t} = \sum_{k=1}^{n_{1t}} Y_{1t,k} - \theta_1 \lambda_1$$

$$\eta_{2t} = \sum_{l=1}^{n_{2t}} Y_{2t,l} - \theta_2 \lambda_2 \quad \text{(4.8)}$$

where $\theta_s$ and $\lambda_s$ are the mean jump size $E(Y_{st,i}) = \theta_s$ and mean number of times of jump $E(n_{st}) = \lambda_s$ for currency $s = 1, 2$. Conditional distribution of $R_t$ given $n_{1t} = i, n_{2t} = j$, and all past information is bivariate normal with mean vector

$$\tau = \begin{pmatrix} \mu_1 + i \theta_1 - \lambda_1 \theta_1 \\ \mu_2 + j \theta_2 - \lambda_2 \theta_2 \end{pmatrix}$$

As $\epsilon_t$ and $\eta_t$ are independent the covariance matrix can be written as $\Lambda = \Omega + \Delta$ where

$$\Omega = \begin{pmatrix} \sigma^2_{1t} & \sigma_{12,t} \\ \sigma_{21,t} & \sigma^2_{2t} \end{pmatrix},$$

$$\Delta = \begin{pmatrix} i \delta^2_1 & \rho_{12} \sqrt{i j} \delta_1 \delta_2 \\ \rho_{12} \sqrt{i j} \delta_1 \delta_2 & j \delta^2_2 \end{pmatrix}.$$
Then we can write the conditional distribution of return $R_t$ given $n_{1t} = i, n_{2t} = j$:

$$f(R_t \mid n_{1t} = i, n_{2t} = j, \Phi_{t-1}) = (2\pi)^{-\frac{3}{2}} |\Lambda|^{-\frac{1}{2}} \exp\{-\frac{1}{2} (R_t - \tau)' \Lambda^{-1} (R_t - \tau)\}.$$ 

Therefore the unconditional (on $n_{1t}$ and $n_{2t}$) density of returns is written as

$$P(R_t \mid \Phi_{t-1}) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} f(R_t \mid n_{1t} = i, n_{2t} = j, \Phi_{t-1}) P(n_{1t} = i, n_{2t} = j \mid \Phi_{t-1}),$$

where $P(n_{1t} = i, n_{2t} = j \mid \Phi_{t-1})$ is the bivariate Poisson distribution function:

$$P(n_{1t} = i, n_{2t} = j \mid \Phi_{t-1}) = e^{-\lambda_1 + \lambda_2 + \lambda_3} \sum_{k=0}^{m} \frac{(\lambda_1 - \lambda_3)^i}{i!} \frac{(\lambda_2 - \lambda_3)^j}{j!} \frac{\lambda_3^k}{k!},$$

where $m = \min(i, j)$. The third parameter $\lambda_3$ is associated with the covariance of the bivariate Poisson distribution. The marginal densities are given by one dimensional Poisson distribution:

$$P(n_{st} = i \mid \Phi_{t-1}) = \frac{e^{-\lambda_s} \lambda_s^i}{i!}, \ s = 1, 2,$$

and the correlation between $n_{1t}$ and $n_{2t}$ is given by

$$\text{corr}(n_{1t}, n_{2t}) = \frac{\lambda_3}{\sqrt{\lambda_1 \lambda_2}}.$$ 

The log likelihood function is given by

$$\ln L = \sum_{t=1}^{T} \ln P(R_t \mid \Phi_{t-1}).$$

We calculate maximum likelihood estimators (MLE) for this model. As is seen the likelihood function $\ln L$ is very complicated and contains about 20 unknown parameters, the maximum likelihood method needs extremely long computer time. Therefore we need to try and get some smart starting values. To do so, we propose two-step method, which is explained in the Appendix. We show the estimated parameters by ML method for the bivariate GARCH jump model for the pairs of currencies, i.e., the Yen and the Australian dollar, the Yen and the Euro, the Yen and the Canadian Dollar, the Yen and the Dollar, in Table 2 where the subscript 1 denotes the Japanese Yen, and 2 the counterpart currency. The estimated parameters are summarized in
Table 2.

Note that $\alpha + \beta$ is nearly one for all cases. This result seems consistent with the volatility persistence after the jump. We also note that $\omega$ is almost always zero.

Table 2: ML estimates of the parameters in the Bivariate GARCH-Jump Model

<table>
<thead>
<tr>
<th></th>
<th>Yen-Australian$</th>
<th>Yen-Euro</th>
<th>Yen-Canadian$</th>
<th>Yen-Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Jump</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.2971</td>
<td>0.3577</td>
<td>0.4024</td>
<td></td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.3182</td>
<td>0.2164</td>
<td>0.2971</td>
<td></td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>0.1582</td>
<td>0.1042</td>
<td>0.0881</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.2306</td>
<td>0.2302</td>
<td>0.2422</td>
<td>5.27E-01</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>-0.0247</td>
<td>-0.0239</td>
<td>-0.0253</td>
<td>-0.0234</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-0.0049</td>
<td>-0.0003</td>
<td>-0.0114</td>
<td>0.0092</td>
</tr>
<tr>
<td>$\delta_1^2$</td>
<td>0.0024</td>
<td>-4.78E-07</td>
<td>-8.67E-07</td>
<td>0.0037</td>
</tr>
<tr>
<td>$\delta_2^2$</td>
<td>-5.99E-08</td>
<td>0.0012</td>
<td>0.00088</td>
<td>-8.25E-08</td>
</tr>
<tr>
<td><strong>GARCH</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>3.54E-05</td>
<td>6.20E-06</td>
<td>1.01E-05</td>
<td>7.11E-06</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.1167</td>
<td>0.0604</td>
<td>0.1208</td>
<td>0.1162</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.8127</td>
<td>0.7489</td>
<td>0.7899</td>
<td>0.4790</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>2.70E-06</td>
<td>1.42E-06</td>
<td>1.52E-06</td>
<td>2.23E-06</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.0309</td>
<td>0.0523</td>
<td>0.0501</td>
<td>0.0316</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.4163</td>
<td>0.7108</td>
<td>0.7313</td>
<td>0.4928</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>-8.99E-07</td>
<td>-9.65E-07</td>
<td>3.53E-07</td>
<td>-1.27E-06</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.0601</td>
<td>0.0562</td>
<td>0.0778</td>
<td>0.0606</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.5817</td>
<td>0.7296</td>
<td>0.7600</td>
<td>0.4859</td>
</tr>
</tbody>
</table>

As is previously defined, the correlation coefficient between $n_{1t}$ and $n_{2t}$ equals $\lambda_3 / \sqrt{\lambda_1 \lambda_2}$, and it can be estimated by $\lambda$s in Table 2. The estimated correlations are given in Table 3.
Table 3: Estimated Correlation Coefficient Between $n_{1t}$ and $n_{2t}$

<table>
<thead>
<tr>
<th></th>
<th>Yen-Australian$</th>
<th>Yen-Euro</th>
<th>Yen-Canadian$</th>
<th>Yen-Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yen-Australian$</td>
<td>0.4032</td>
<td>0.5145</td>
<td>0.3745</td>
<td>0.2548</td>
</tr>
</tbody>
</table>

5 Model performance

In this section we compare the performance of the models considered in this paper. The models are evaluated by $\alpha\%$ Value-at-Risk (VaR) Threshold. If $\alpha\%$ of the observed return $r_t$ exceed $\alpha\%$ VaR threshold we can say that the model is well performed. VaR threshold at time $t$ is calculated by $c_\alpha \times \sqrt{\hat{h}_t}$ where $c_\alpha$ is the percentile point of the assumed distribution of return and $\hat{h}_t$ is calculated from the estimated variance equation:

$$\hat{h}_t = \hat{\omega} + \hat{\alpha}_\epsilon \hat{\epsilon}_t^2 + \hat{\beta}\hat{h}_{t-1},$$

where the parameters were estimated by using all observations. The $c_\alpha \times \sqrt{\hat{h}_t}$ are graphed in Fig.7 - Fig.12. Theoretically $c_\alpha$ should be $\alpha$-percentile of the normal distribution, because we assumed normality in our model. But we also use $\alpha$-percentile of t-distribution with 3 degree of freedom just for trial. If $\alpha$-percentile of t-distribution is fitted well, it means that $\epsilon$ in the model should have been assumed to follow t distribution.

We count the number of observation which exceeds or violates 1% VaR threshold, and the likelihood ratio. Table 4 shows percentage of violation. From this table we note the following points:

1) In uni-variate GARCH(1,1) models, we see that the best case is the uni-variate
GARCH(1,1) with jumps for the Yen. For other currencies the performance are not very well in both models with and without jumps.

(2) In bi-variate GARCH(1,1) models for the Yen, GARCH(1,1) model with jumps is better than the model without jumps. For other currencies the jump model is relatively better than the mode without jumps.

Table 4: Evaluating VaR Thresholds

<table>
<thead>
<tr>
<th></th>
<th>Currency</th>
<th>Proportion of Violation under N</th>
<th>Proportion of Violation under t</th>
<th>Likelihood Ratio under N</th>
<th>Likelihood Ratio under t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uni-Variate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without jump</td>
<td>YEN</td>
<td>1.81%</td>
<td>0.069%</td>
<td>0.0221</td>
<td>2.05E-05</td>
</tr>
<tr>
<td>with jump</td>
<td>YEN</td>
<td>1.32%</td>
<td>0.139%</td>
<td>0.5095</td>
<td>0.0002</td>
</tr>
<tr>
<td>Bivariate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without jump</td>
<td>YEN</td>
<td>21.25%</td>
<td>12.431%</td>
<td>NA</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>AU$</td>
<td>31.25%</td>
<td>21.736%</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>with jump</td>
<td>YEN</td>
<td>2.08%</td>
<td>0.208%</td>
<td>0.0015</td>
<td>0.0012</td>
</tr>
<tr>
<td></td>
<td>AU$</td>
<td>26.32%</td>
<td>12.778%</td>
<td>NA</td>
<td>0</td>
</tr>
</tbody>
</table>

N: Normal distribution, t: t distribution.
6 Concluding remarks

When the Chinese Yuan was revaluated on the 21 July 2005 a big jump was observed in 1-minute high frequency time series of returns of major currencies: the Japanese Yen, the Australian dollar, the Canadian dollar, the Euro, and the Pound sterling against the US dollar. And the jump was followed by large volatility for about 6 hours like a ripple created by a stone. We attempt to describe this phenomena by a uni-variate and a bi-variate GARCH(1,1) model with or without correlated Poisson jump. We estimated these models by ML method and evaluated the estimated models by using Value-at-Risk. As a result we note that although there is not a model which is uniformly superior to other models, as far as the Yen is concerned GARCH-Jump model is better than GARCH model without jump.
References


Appendix: Initial values for ML estimation

To get the initial values for ML estimation we use two-step method, which consists of the following steps: (1) Step 1: Extract observations for jumps based on a criterion $c = 0.05$, and estimate parameters for jump part by descriptive statistics under a criterion $c$ by assuming that the number of jump per unit time interval follows Poisson distribution. (2) Step 2: Estimate GARCH parameters after deleting the jumps from the data.

We show the estimated parameters by 2-step method for the bivariate GARCH jump model for the pairs of currencies, i.e., the Yen and the Australian dollar, the Yen and the Euro, the Yen and the Canadian Dollar, the Yen and the Dollar, in Table 5 where the subscript 1 denotes the Japanese Yen, and 2 the counterpart currency. Two-step method is justified because of the assumption of independence between GARCH part and Jump component.
Table 5: Two-Step estimates of the parameters in the Bivariate GARCH-Jump Model

<table>
<thead>
<tr>
<th></th>
<th>Yen-Australian$</th>
<th>Yen-Euro</th>
<th>Yen-Canadian$</th>
<th>Yen-Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jump</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.3229</td>
<td>0.3229</td>
<td>0.3229</td>
<td>0.3229</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.2118</td>
<td>0.1979</td>
<td>0.1667</td>
<td>0.2083</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>0.0944</td>
<td>0.1059</td>
<td>0.0764</td>
<td>0.1231</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>-0.0223</td>
<td>-0.0223</td>
<td>-0.0223</td>
<td>-0.0223</td>
</tr>
<tr>
<td>$\theta_2$</td>
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Fig. 1

The upper line shows the spot exchange rate for the Japanese Yen against US Dollar from 12:00 a.m. on July 21, 2005 to 11:59 p.m. on July 22, 2005 and its values are displayed on the vertical axis on the left. The lower line presents the returns for the currency. The vertical axis on the right-hand side shows the values of the returns.
Fig. 2
Australian Dollar / US Dollar and Returns

Fig. 3
Fig. 5
Sample pass simulated by the estimated model

Fig. 6
Fig. 7  Returns and VaR Thresholds of YEN Calculated by Full Sample Uni-GARCH
Fig. 8 Returns and VaR Thresholds of YEN Calculated by Full Sample Unii-
Jump - GARCH
Fig. 9 Returns and VaR Thresholds of YEN Calculated by Full Sample BV-GARCH (YEN, AU$)
Fig. 10 Returns and VaR Thresholds of AU$ Calculated by Full Sample BV-
GARCH (YEN, AU$)
Fig. 11  Returns and VaR Thresholds of YEN Caculated by Full Sample BV.
Jump _ GARCH (YEN, AU$)
Fig. 12 Returns and VaR Thresholds of AU$ Calculated by Full Sample BV
Jump - GARCH (YEN, AU$)